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GENERALIZED PERFORMANCE SELECTION CHARTS FOR SINGLE-ENGINE PURSUIT AIRPLANES

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MEMORANDUM REPORT

for the

Army Air Forces, Materiel Command

GENERALIZED PERFORMANCE SELECTION CHARTS FOR SINGLE-ENGINE PURSUIT AIRPLANES

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SUMMARY

The NACA has carried out an investigation of the effect of wing loading, power loading, and aspect ratio on the performance of pursuit airplanes. The study is based on data combined from specifications, wind-tunnel and flight tests of various modern fighter airplanes, and the results are presented on graphs that show the performance that may be obtained by airplanes having an aerodynamic cleanness that is approximately the best found in present production airplanes.

For the purpose of this report, the charts are prepared for airplanes powered by one 2000-horsepower radial air-cooled engine, but are placed on coordinates that allow their use for airplanes of different power for the determination of trends.

INTRODUCTION

The NACA in cooperation with the military services has made a study in which bomber airplane performance has been graphically related to bomber parameters. These studies were based upon statistical information obtained on existing military bomber types. The present report continues this study in regard to single-engine pursuit airplane performance based upon statistical information on military pursuit airplanes.

In reference 1 it was shown that airplane parameters are divided into two groups:

Primary (dimensional)	Secondary (efficiency)
Altitude	Engine economy
Power	Aerodynamic efficiency
Gross weight	Structural efficiency
Wing area	

The reports of references 2, 3, 4, 5, and 6 related bomber performance to the primary parameters while the secondary parameters were held constant. Reference 1 examined the effect of magnified variations of the secondary or efficiency parameters while the primary parameters were held constant. The present report relates the primary parameters of pursuit airplanes to their performance while the secondary or efficiency parameters are held constant.

For this analysis the power is taken as 2000 brake horsepower and the altitude, gross weight, and wing area are varied. The efficiency parameters are selected to be comparable to latest production pursuit airplanes; therefore the charts are useful in determining approximate performances that may be realized today. The performance characteristics that are related to the above parameters are: speed, rate of climb, take-off distance, rate of roll, radius of turn, and time to turn. The report is roughly divided into two parts: (1) the effect of wing loading, power loading, and dimensions on the general trends of performance of pursuit airplanes and (2) the effect of varying the wing area and aspect ratio of a given airplane with a fixed load, permitting changes in the wing and power loadings according to those required for structural weight variations. The pursuit airplane differs from the bomber in that its gasoline and bomb loads are fixed at a minimum to permit high rate of climb and great maneuverability; therefore, the second part of the report is made a necessary addition since the change in structural weight cannot alter this load as was done in the bomber study.

The main purpose of this report is to show the trends of performance in a convenient graphical form that may be used without laborious computations. The results of the analysis are presented in the form of charts which are discussed in the body of the report. The assumptions upon which the charts are built are given in appendix A, the formulas used are given in appendix B, and a discussion of the limitations of the charts is given in appendix C.

The charts are arranged as follows:

Figure	Subject	Altitude (ft)	Coordinate system
1(a)	Composite selection chart	sea level	W/P versus W/S
1(b)	------------do.------------	15,000	Do.
1(c)	------------do.------------	25,000	Do.
1(d)	------------do.------------	40,000	Do.

Figure	Subject	Altitude (ft)	Coördinate system
2(a)	Take-off run	sea level	W/P versus W/S
2(b)	-----------do.-----------	5,000	Do.
3(a)	Rate of climb	sea level	Do.
3(b)	-----------do.-----------	15,000	Do.
3(c)	-----------do.-----------	25,000	Do.
3(d)	-----------do.-----------	40,000	Do.
4(a)	High speed	sea level	Do.
4(b)	-----------do.-----------	15,000	Do.
4(c)	-----------do.-----------	25,000	Do.
4(d)	-----------do.-----------	40,000	Do.
5(a)	Radius of turn	sea level	Do.
5(b)	-----------do.-----------	15,000	Do.
5(c)	-----------do.-----------	25,000	Do.
5(d)	-----------do.-----------	40,000	Do.
6(a)	Time to turn	sea level	Do.
6(b)	-----------do.-----------	15,000	Do.
6(c)	-----------do.-----------	25,000	Do.
6(d)	-----------do.-----------	40,000	Do.
7	Weight variation	any alt.	Do.
8	Rolling velocity	---do.---	p' versus b, and V_{mph}
9	Rolling velocity with a 30-pound stick force	all alt.	Do.
10(a)	The effect of aspect ratio on power loading (Constant loads)	any alt.	W/P versus R
10(b)	The effect of aspect ratio on take-off run (Airplane A)	sea level and 5,000	s versus R
10(c)	The effect of aspect ratio on rate of climb (Airplane A)	all alt.	V_c versus R
10(d)	The effect of aspect ratio on high speed (Airplane A)	---do.---	V_{mph} versus R
10(e)	The effect of aspect ratio on radius of turn (Airplane A)	---do.---	r versus R
10(f)	The effect of aspect ratio on time to turn (Airplane A)	---do.---	t versus R
10(g)	The effect of aspect ratio on rolling (Airplane A)	---do.---	p' versus R
11(a)	The effect of wing loading on take-off run (Airplane A)	sea level	s versus W/S

Figure	Subject	Altitude (ft)	Coordinats system
11(b)	The effect of wing loading on rate of climb (Airplane A)	all alt.	V_c versus W/S
11(c)	The effect of wing loading on high speed (Airplane A)	---do.---	V_{mph} versus W/S
11(d)	The effect of wing loading on radius of turn (Airplane A)	---do.---	r versus W/S
11(e)	The effect of wing loading on time to turn (Airplane A)	---do.---	t versus W/S
11(f)	The effect of wing loading on rolling (Airplane A)	---do.---	p' versus W/S
11(g)	The effect of wing loading on rolling with a 30-pound stick force (Airplane A)	---do.---	Do.

SYMBOLS

a	acceleration, feet per second per second
b	wing span, feet
c	speed of sound, feet per second
C_D	drag coefficient
C_{D_i}	induced-drag coefficient, $\frac{C_L^2}{e \pi R}$
C_{D_o}	profile-drag coefficient
C_L	lift coefficient
D	drag, pounds
D_w	landing-gear drag, pounds
e	span efficiency factor
F	frontal area of fuselage, square feet
K	experimental constant

L	lift, pounds
L/D	lift to drag ratio
M	Mach number, V/c
P	engine brake horsepower
p	rate of roll, radians per second
p'	rate of roll, degrees per second
P_c	excess brake horsepower for climbing
P_j	engine cooling thrust horsepower
P_o	thrust horsepower required for pumping engine cooling air
q	dynamic pressure of the air stream, $\left(\frac{1}{2}\rho V^2\right)$, pounds per square foot
q_1	q corrected for compressibility, $\left[\frac{1}{2}\rho V^2\left(1 + \frac{1}{4}M^2 + \ldots\right)\right]$, pounds per square foot
r	radius of turn, feet
r_s	minimum radius of steady turn, feet
r_a	minimum radius of accelerated turn, feet
R	aspect ratio, b^2/S
s	take-off distance, feet
S	wing area, square feet
t	time to turn, seconds
T	propeller thrust, pounds
T_o	temperature, °Rankine
V	airplane speed, feet per second
V_c	rate of climb, feet per minute

W gross weight, pounds

η propeller efficiency

ρ air density, slugs per cubic foot

μ rolling friction coefficient

PRESENTATION OF FIGURES

Selection Charts. - Performance selection charts for single-engine pursuit airplanes powered by a 2000-horsepower engine are given for four altitudes (sea level, 15,000 feet, 25,000 feet, and 40,000 feet) in figure 1. These charts are drawn on the coordinates of the airplane wing and power loading. They present performance curves of speed, rate of climb, take-off distance, minimum radius of steady turn at constant altitude, and minimum time to turn 360°. Using the weight formula of appendix B, the gross weight of many airplanes is computed for constant internal loads (the internal loads are considered to be the gross weight minus structural weight); the loads are expressed as a power loading, W_5/P, and are plotted on the selection charts to indicate the available airplane power loading for a given load and wing loading. A combination of the weight curves and the performance curves permit the determination of the effect of changing the wing area of a given airplane on its performance. Each of these performance and weight curves are given in more detail as a function of wing and power loading in figures 2 to 7.

The values of the efficiency parameters used in the construction of these charts correspond to the best present-day production airplanes and therefore they are useful in obtaining a rough approximation to the available performance of pursuit airplanes with a given set of primary parameters. For example, if it is desired to determine approximately the performance of a pursuit airplane with a wing loading of 40 pounds per square foot and a power loading of 6 pounds per horsepower with an engine supercharged to 25,000 feet altitude, the performance may be read either from figure 1(c) or in more detail in figures 2 to 8. For airplanes having 2000 horsepower we have $W = 12,000$, $S = 300$, and $b = 41.5$.

The performance estimates obtained are:

1. High speed, 435 miles per hour
2. Rate of climb at 25,000 feet altitude, 2250 feet per minute
3. Take-off run at sea level, 1270 feet
4. Minimum radius of steady turn at 25,000 feet altitude, 2250 feet
5. Minimum time to turn 360° at 25,000 feet altitude, 43 seconds
6. Rate of roll at 400 miles per hour at 25,000 feet altitude, 68° per second

Rate of roll. - Unlike the other performances, rate of roll is not a function of engine power or airplane weight and therefore it is not useful to plot this performance on the coordinates of wing and power loading. The actual rate of roll of an airplane is dependent upon many factors that cannot be simply represented in chart form, for example, the aerodynamic and mechanical balance of the ailerons, the aileron effectiveness, the wing stiffness, and moment of inertia about the rolling axis. However, certain general trends of this performance characteristic may be conveniently represented in three-dimensional graphs, if representative values of the above effects are taken from experience on airplanes.

The geometric relationship of the helical path of an airplane in a steady roll may be written in the form

$$\frac{pb}{2V} = K$$

By assigning values to K, this relationship may be shown as surfaces on three-dimensional plots. It has been found that the value of this constant for a representative pursuit airplane is $K = 0.084$. This surface is plotted in figure 8 on the corrdinates of wing span in feet, airplane forward speed in miles per hour, and rolling velocity in degrees per second. To obtain the rolling velocity for any other value of K, the values of rolling velocity from figure 8 may be multiplied by the ratio of values of K.

This relationship must be modified in the actual airplane by the stick force the pilot can exert. If the values obtained on the same representative pursuit airplane are used to fix the degree of aerodynamic balance, a new surface may be placed on these three-dimensional plots by fixing values of altitude and aspect ratio. (See appendix B for methods of analysis.) Figure 9 shows these double surface plots for various altitudes with an aspect ratio of 5.75. These surfaces combined with the planes through the coordinate axes form a volume. Several planes are passed through this volume and are shown shaded in each part of figure 9 in order to show the rolling performance of representative airplanes as a function of speed.

Since the time required for an airplane to reach its maximum rate of roll is very short, particularly at the high speeds at which rate of roll seems important, the initial period of acceleration is neglected and only steady rolling is considered. It is worth while to understand the limitations on the airplane under these conditions. At low airspeeds, the pilot can deflect the ailerons fully and roll at a given helix angle in which case the rate of roll varies directly with speed and inversely with span, being independent of wing area. At very high airspeeds the limitation on stick force determines the rate of roll; for airplanes having a constant aspect ratio the maximum rolling velocity varies inversely with speed and inversely as the fourth power of the span or inversely as the square of the wing area.

Effect of aspect ratio of airplane A . - The change of performance due to a change in the aspect ratio of the wing of a given airplane is shown in figure 10. The airplane chosen for the example is shown on figure 1 as airplane A. It has a power loading of 6 pounds per horsepower and a wing loading of 40 pounds per square foot, which for this study corresponds to a gross weight of 12,000 pounds with 300 square feet of wing area.

The variation in the power loading due to the change in structural weight with aspect ratio is given in figure 10(a). In the study of the other performance characteristics it should be remembered that this change in structural weight increases the power loading at high aspect ratio. The optimum aspect ratio for any performance is obtained by the best balance between this structural effect and the effect of aerodynamic considerations. A study of figure 10(c) showing the effect of aspect ratio on rate of climb should suffice to explain this type of chart. In addition to airplane A, two other arbitrary airplanes (A_5 and A_6) are spotted on the figure. It is seen that airplane A has almost the optimum aspect ratio for climbing at 25,000 feet. If the aspect ratio is increased to that of airplane A_5, the rate of climb is less because the extra structural weight more than offsets any gain in aerodynamic efficiency. If the aspect ratio is reduced to that of airplane A_6, the rate of climb is less because the decrease in aerodynamic efficiency more than offsets the effect of lighter structural weights.

Effect of design wing loading on performance. - The change of performance due to a change in wing area of airplane A is given in figure 11. The power loading based on internal loads only (no structural weights) is kept constant at 4.03. The dashed line in figure 7 gives the connection between the power loading of the airplane and the resulting wing loading as the wing area is varied. It should be remembered for the purpose of this section that the airplane with a light wing loading is heavier than the airplane with a high wing loading.

A study of figure 11(b) is helpful in understanding the figures showing the effect of wing loading on performance. One conclusion easily drawn is that small changes in wing area (and hence wing loading) have very little effect on rate of climb at sea level, but at high altitudes a low wing loading is helpful.

PERFORMANCE TRENDS

In order to study the trends in performance as affected by changes in loads, wing area, and power loading, several airplanes are spotted on figure 7. The performance of each is tabulated in the following sections, for airplanes having 2000 horsepower. The basis of comparison is airplane A with $W = 12{,}000$, $S = 300$, $R = 5.75$. The rate of roll is given at 400 miles per hour true airspeed, and all take-off runs are at sea level.

<u>The effect of removing a 1000-pound load</u>. - Airplane A_1 is obtained by removing a load of 1000 pounds from airplane A thus obtaining $W = 11{,}000$, $S = 300$, $R = 5.75$. The following tables can be used for comparison of airplanes A and A_1.

	Wing loading, lbs/sq ft	Power loading, lbs/hp	Span, feet	Take-off run, feet	Rate of climb, feet/minute	High speed, mph	Rad. of accel. turn, feet	Rad. of steady turn, feet	Time to turn 360°, seconds	Rate of roll, deg/sec
Comparison of A and A_1 at sea level										
Airplane A	40	6.0	41.5	1270	3075	355	420	580	19.2	31
Airplane A_1	36.7	5.5	41.5	1050	3500	355	385	500	17.5	31
A_1/A	.92	.92	1.0	.83	1.14	1.0	.92	.87	.91	1.0
Comparison of A and A_1 at 15,000 feet altitude										
Airplane A	40	6.0	41.5		2625	400	670	1025	28	49
Airplane A_1	36.7	5.5	41.5		3000	400	610	900	25	49
A_1/A	.92	.92	1.0		1.15	1.0	.92	.87	.91	1.0
Comparison of A and A_1 at 25,000 feet altitude										
Airplane A	40	6.0	41.5		2250	435	935	2250	43	68
Airplane A_1	36.7	5.5	41.5		2750	435	850	1900	39	68
A_1/A	.92	.92	1.0		1.22	1.0	.92	.85	.89	1.0
Comparison of A and A_1 at 40,000 feet altitude										
Airplane A	40	6.0	41.5		1625	510	1700	4750	70	125
Airplane A_1	36.7	5.5	41.5		2100	510	1560	4000	61	125
A_1/A	.92	.92	1.0		1.3	1.0	.92	.84	.87	1.0

The effect of designing for a 1000-pound lighter load. - Airplane A_2 is obtained by designing airplane A for a load 1000 pounds less than the original design. This gives an airplane similar to airplane A_1 except that the structural weight is less. From the data of figure 7 we obtain $W = 10{,}600$, $S = 300$, $R = 5.75$. The following tables list the data on airplanes A and A_2.

	Wing loading, lbs/sq ft	Power loading, lbs/hp	Span, feet	Take-off run, feet	Rate of climb, feet/ minute	High speed, mph	Rad. of accel. turn, feet	Rad. of steady turn, feet	Time to turn 360°, seconds	Rate of roll, deg/sec
Comparison of A and A_2 at sea level										
Airplane A	40	6.0	41.5	1270	3075	355	420	580	19.2	31
Airplane A_2	35	5.3	41.5	950	3750	355	370	460	16.5	31
A_2/A	.88	.88	1.0	.75	1.22	1.0	.88	.79	.86	1.0
Comparison of A and A_2 at 15,000 feet altitude										
Airplane A	40	6.0	41.5		2625	400	670	1025	28	49
Airplane A_2	35	5.3	41.5		3300	400	580	825	23.5	49
A_2/A	.88	.88	1.0		1.26	1.0	.88	.80	.85	1.0
Comparison of A and A_2 at 25,000 feet altitude										
Airplane A	40	6.0	41.5		2250	435	935	2250	43	68
Airplane A_2	35	5.3	41.5		3000	435	815	1750	37	68
A_2/A	.88	.88	1.0		1.33	1.0	.88	.78	.84	1.0
Comparison of A and A_2 at 40,000 feet altitude										
Airplane A	40	6.0	41.5		1625	510	1700	4750	70	125
Airplane A_2	35	5.3	41.5		2400	510	1500	3600	58	125
A_2/A	.88	.88	1.0		1.48	1.0	.88	.76	.83	1.0

<u>The effect of change in design power loading</u>. - This section shows the effect of designing an airplane for a lower power loading with the same wing loading. Airplane A_3, the airplane used here for an example, is obtained by designing airplane A for a load 1000 pounds less than normal and then decreasing the wing area (and hence the structural weight) until the wing loading is the same as that of airplane A. From figure 7 we can estimate W = 10,400, S = 260, R = 5.75. The following tables list the details of airplanes A and A_3.

	Wing loading, lbs/sq ft	Power loading, lbs/hp	Span, feet	Take-off run, feet	Rate of climb, feet/minute	High speed, mph	Rad. of accel. turn, feet	Rad. of steady turn, feet	Time to turn 360°, seconds	Rate of roll, deg/sec
Comparison of A and A_3 at sea level										
Airplane A	40	6.0	41.5	1270	3075	355	420	580	19.2	31
Airplane A_3	40	5.2	38.6	1070	3750	365	420	530	17.8	41
A_3/A	1.0	.87	.89	.84	1.22	1.03	1.0	.93	.93	1.32
Comparison of A and A_3 at 15,000 feet altitude										
Airplane A	40	6.0	41.5		2625	400	670	1025	28	49
Airplane A_3	40	5.2	38.6		3250	410	670	950	25.5	65
A_3/A	1.0	.87	.89		1.24	1.03	1.0	.91	.91	1.32
Comparison of A and A_3 at 25,000 feet altitude										
Airplane A	40	6.0	41.5		2250	435	935	2250	43	68
Airplane A_3	40	5.2	38.6		2900	450	935	2000	39	91
A_3/A	1.0	.87	.89		1.29	1.03	1.0	.89	.89	1.32
Comparison of A and A_3 at 40,000 feet altitude										
Airplane A	40	6.0	41.5		1625	510	1700	4750	70	125
Airplane A_3	40	5.2	38.6		2200	525	1700	4100	61	150
A_3/A	1.0	.87	.89		1.35	1.03	1.0	.87	.87	1.2

<u>The effect of design wing loading.</u> - To show the effect of designing an airplane for a different wing loading, it is necessary to design for the same internal load. Airplane A_4 is the airplane obtained by designing airplane A for a wing loading of 35 instead of 40; it is obtained by moving along the constant load line ($W_5/P = 4.03$) in figure 7 until the desired point is reached. From the given wing loading, and the power loading indicated in figure 7, we find that airplane A_4 has $W = 12,000$, $S = 349$, $R = 5.75$. Further results are presented in the following tables.

	Wing loading, lbs/sq ft	Power loading, lbs/hp	Span, feet	Take-off run, feet	Rate of climb, feet/minute	High speed, mph	Rad. of accel. turn, feet	Rad. of steady turn, feet	Time to turn 360°, seconds	Rate of roll, deg/sec
Comparison of A and A_4 at sea level										
Airplane A	40	6.0	41.5	1270	3075	355	420	580	19.2	31
Airplane A_4	35	6.1	44.8	1140	3100	340	370	510	17.7	23
A_4/A	.88	1.02	1.08	.90	1.01	.96	.88	.88	.92	.74
Comparison of A and A_4 at 15,000 feet altitude										
Airplane A	40	6.0	41.5		2625	400	670	1025	28	49
Airplane A_4	35	6.1	44.8		2675	385	580	910	25.5	36
A_4/A	.88	1.02	1.08		1.02	.96	.88	.88	.92	.74
Comparison of A and A_4 at 25,000 feet altitude										
Airplane A	40	6.0	41.5		2250	435	935	2250	43	68
Airplane A_4	35	6.1	44.8		2325	420	815	2000	40	50
A_4/A	.88	1.02	1.08		1.03	.96	.83	.88	.92	.74
Comparison of A and A_4 at 40,000 feet altitude										
Airplane A	40	6.0	41.5		1625	510	1700	4750	70	125
Airplane A_4	35	6.1	44.8		1750	495	1500	4200	65	93
A_4/A	.88	1.02	1.08		1.08	.96	.88	.88	.92	.74

CONCLUSIONS

1. Take-off distance is adversely affected by increased power loading or wing loading.

2. Rate of climb is increased by a decrease in the power loading of the airplane. The effect of wing loading is negligible at sea level, but light wing loadings are advantageous at high altitudes.

3. The maximum speed increases with wing loading and decreases with power loading in the normal range of values used for pursuit airplanes. The maximum speed of pursuit airplanes is almost unaffected by overloading in the normal range.

4. The minimum radius of steady turn at constant altitude is adversely affected by increased power loading or wing loading in the normal range encountered on modern pursuit airplanes.

5. The minimum radius of accelerated turn increases with wing loading. It is not affected by power loading.

6. The time to turn at constant altitude is adversely affected by increased wing and power loading.

7. At high forward speed where the aileron deflection is determined by the stick force, the rate of roll decreases with forward velocity and wing area. For this condition the rate of roll is unaffected by aspect ratio. At low forward speeds where the pilot can give the ailerons full deflection, the rate

of roll increases with forward speed and decreases with wing span. In this case low aspect ratio is advantageous.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., February 6, 1943.

APPENDIX A

Airplane Characteristics

Airplane characteristics change because of improvements in engines, materials, and aerodynamic design. The airplanes that form the basis of the report represent present design. A study of information obtained from the Materiel Command Liaison Office at Langley Field and from reports on pursuit airplanes tested in the NACA full-scale tunnel led to the assumption of a wing and tail drag coefficient of 0.0110 based on wing area, and also a fuselage drag coefficient of 0.08 based on frontal area. This gives $C_D = 0.0110 - 0.0172^{-\frac{1}{2}} + \frac{0.08F}{S} + \frac{{C_L}^2}{e\pi R}$, the second term representing a correction for the wing area enclosed by the fuselage. The horsepower required for pumping the engine cooling air, P_o, and the thrust power, P_j, obtainable from the heated air leaving the cowling, are calculated for the high-speed condition, but P_j is neglected at all other speeds.

Since high altitude operation required bigger ducts and more supercharging equipment, it is necessary to increase the effective frontal area in the airplanes designed for high altitude. In a similar manner it is necessary to change the propeller efficiency and power required for cooling with altitude. The following table lists the simultaneous values used.

TABLE I - Pertinent Assumptions

F, sq ft	P_o, hp	Altitude (ft)	η_{turn}	η_{climb}	$\eta_{high\ speed}$	R
23.5	[illegible]	0	0.0039V	0.80	0.85	5.75
23.5	[illegible]	15,000	.0032V	.78	.82	5.75
24.0	165	25,000	.0021V	.78	.82	5.75
25.0	185	40,000	.0018V	.78	.82	5.75

Cooling Power

The power required for engine cooling is estimated from reference 7 with 50 percent extra fin width for the calculations at 40,000 feet. Power for pumping air for the intercooler is found from reference 6. The total cooling power is tabulated above as P_o. The thrust possible from the hot engine cooling air is estimated from reference 9 to be

$$P_j = \frac{561V^2}{25950T_o + V^2}$$

Fuselage

It is assumed that the effective cross section of the fuselage is independent of the weight of the airplane since the cross section will be influenced mainly by the engine diameter and the altitude the airplane must be supercharged to. These items set the minimum diameter and there seems to be little advantage to increasing above this minimum.

Span Efficiency

The span efficiency factor is assumed to be 0.8 for all lift coefficients less than 1.5, thereafter decreasing linearly to 0.6 at a lift coefficient of 2.5.

Aspect Ratio

An aspect ratio of 5.75 is used in all the calculations except those showing the effect of aspect ratio.

Landing Gear

During take-off calculations the additional air drag caused by the lowered landing gear is considered. The extra drag is based on tests indicating that the drag of tires accounts for approximately 70 percent of the total landing-gear air drag. The airplanes are considered as being supported by 10-ply high-pressure tires in which case the tire frontal area and hence drag will vary as the airplane weight. The drag can be expressed as $D_W = 1.3 \times 10^{-7} W V^2$.

A value of rolling friction coefficient of 0.05 is used; this value is to be expected on fields with short grass.

Propeller Efficiency and Thrust

Propeller sizes and efficiencies were investigated with the help of reference 10 for 15,000, 25,000, and 40,000 feet altitude with propeller tips operating at the speed of sound in the high-speed condition. In order to keep the trends in performance correct it is decided to vary propeller efficiency with altitude and speed in a manner to be expected for

2000-horsepower propellers of small diameter and high tip speed. The values used for the efficiency are listed in table I. Since the calculations for turning are at a low speed, it is assumed that the efficiency varies directly with speed up to the maximum efficiency.

APPENDIX B

PERFORMANCE CALCULATIONS

Take-off Run

Take-off run is considered as the distance required for the airplane to accelerate to a speed at which it can leave the ground with a lift coefficient of 1.0. The propeller designed for use at 15,000 feet is used in the take-off calculations because it gives results that will be more conservative than the propellers designed for higher altitude. The thrust for the propeller is matched closely up to 300 feet per second by the curve:

$$T = T_o - K\ V^2$$

at sea level the thrust is taken as $T = 5100 - 0.0179\ V^2$ and at 5000 feet altitude $T = 4470 - 0.0154\ V^2$. Equating the summation of the forces to the mass times the acceleration

$$T - \mu W - \frac{\rho}{2} V^2\ SC_{D_o} - D_w = \frac{W}{g} a$$

where μ is taken as 0.05, $C_{D_o} = 0.0110 - 0.017S^{-\frac{1}{2}} + \frac{0.08F}{S}$, $D_w = 1.3 \times 10^{-7}\ W\ V^2$. Integrating and solving for the distance required we have an equation of the form

$$s = \frac{K_6\ (1.15)}{K_5} \quad \log_{10} \frac{1}{1 - \frac{K_5\ V^2}{K_4}}$$

where

$K_4 = T_o - 0.05W$, $K_5 = (-0.0110S + 0.017S^{\frac{1}{2}} - 1.88)\ \frac{\rho}{2} - 1.3 \times 10^{-7} W - K$,

$K_6 = \frac{W}{32.17}$, $\quad V^2 = \frac{W}{\frac{\rho}{2}\ SC_L}$

Figure 2 is plotted for the case of all airplanes being pulled to a lift coefficient of 1 when the airplane has sufficient speed for the take-off. Points on the graph are obtained by substituting the corresponding values of W and S in the above equations.

Rate of Climb

An investigation of the horsepower available and horsepower required shows that the rate of climb at maximum L/D and the maximum rate of climb are very nearly the same for all airplanes and hence all rates of climb are calculated in the maximum L/D condition by use of the following equations. Equating the horsepower available with that required

$$\eta P = P_o + \frac{2\sqrt{(2/\rho)}\ C_{D_o}^{\frac{1}{4}}\ W^{\frac{3}{2}}}{550\ (e\pi R)^{\frac{3}{4}}\ S^{\frac{1}{2}}} + \eta P_c$$

$$V_c = \frac{33000\ \eta P_c}{W}$$

High Speed

Equating horsepowers in the high-speed condition

$$\eta P + P_j = P_o + \frac{q_1\ SV}{550}\left(0.0110 - 0.017S^{-\frac{1}{2}} + \frac{0.08F}{S} + \frac{C_L^2}{e\pi R}\right)$$

Substituting $\frac{W^2}{S^2 q_1{}^2 e\pi R} = \frac{C_L{}^2}{e\pi R}$ and solving for W

$$W = \sqrt{e\pi R\ S q_1{}^2 \left[550\ \frac{(\eta P - P_o + P_j)}{q_1 V} - 0.0110S + 0.0170S^{\frac{1}{2}} - 0.08F\right]}$$

Substituting various values of S for each value of V we obtain curves of constant velocity as in figure 4.

The values of q_1 are obtained from a chart showing the dynamic pressure corrected for compressibility as a function of altitude and speed. In the absence of a chart of this type, the dynamic pressure can be calculated by the relation

$$q_1 = \frac{1}{2}\rho V^2 \left(1 + \frac{1}{4} M^2 + \ldots\right)$$

Minimum Radius of Steady Turn and Minimum Radius of Accelerated Turn

The maneuver designated in this report as minimum radius of steady turn is the minimum radius of turn that the airplanes can make without changing their altitude or speeds. Equating powers

$$550\ (\eta P - P_o) = \frac{\rho}{2} S V^3 \left(C_{D_o} + \frac{C_L{}^2}{e\pi R}\right)$$

If we have the vertical component of the lift equal to the weight of the airplane, the horizontal component gives an acceleration toward the center of the turning circle.

$$\sqrt{L^2 - W^2} = \frac{W}{g}\frac{V^2}{r}$$

It follows that

$$r = \frac{WV^2}{g\sqrt{\left(\frac{\rho}{2}\ SV^2C_L\right)^2 - W^2}}$$

Substituting a given value of W and S we obtain a value of V and r for each C_L value used. If we plot r versus V for an airplane, the minimum ordinate of the curve represents the minimum radius of turn.

The equations used can be differentiated and solved for the minimum radius but the assumptions are such that four formulas of considerable complexity are required for each altitude, each applying to some range of lift coefficients and velocities.

For airplanes in a vertical bank, the above formula can be simplified to $r = \frac{W}{S}\ \frac{2}{g\rho C_{L_{max}}}$

Minimum Time to Turn

In finding the minimum time to turn, the calculations are made for a turn of 360°. The time for turning smaller angles can be found by multiplying the time indicated on the charts by the corresponding fraction of a complete turn. Given that

$$t = \frac{2\pi r}{V}$$

we see that the time to turn is a minimum when $\frac{r}{V}$ is a minimum. This corresponds to the point of tangency of a line from the origin to the curve of r versus V from the preceding part.

Obviously the line from the origin becomes tangent to the curve of r versus V at a higher speed than the minimum point of the curve, hence the minimum time to turn is always at a higher speed than the speed for minimum radius of turn except where both maneuvers occur at the stall.

The Effect of Aspect Ratio

An example (airplane A: $W/P = 6$, $W/S = 40$, $R = 5.75$) is chosen for an investigation of the effect of varying the aspect ratio on performance. The equations used are similar to those already given except that considerable simplification of the formulas can be made. Wing area is assumed constant but weight and hence power loading and wing loading vary as calculated by the formulas under the heading, Weight Variation.

The Effect of Wing Loading

Airplane A has its wing area varied, keeping aspect ratio constant at $R = 5.75$. Using the new weights as calculated by the weight variation formulas, the wing loading and corresponding performance is computed by previous methods.

Weight Variation

It is assumed that the weight of the wing can be calculated by the wing weight formula,

$$W_1 = \frac{W - 0.75(W_2) - 0.5(W_3)}{\dfrac{Kt}{f\,R^{\frac{3}{2}}\,S^{\frac{1}{2}}} + 1}$$

W_2, the alighting gear, is taken as 0.08W; W_3, the distributed load, is considered as 1800 pounds for 2000-horsepower pursuit airplanes; K is assumed to be 100,000; t, the wing thickness ratio, is taken as 0.16; f, the ultimate load factor, is 12.

It is assumed that the weight of the tail is 13 percent of the weight of the wing, and the fuselage is 10 percent of the gross weight. The internal concentrated loads of the airplane are considered as W_4. Combining the parts of the airplane

$$W = 0.10W + 0.08W + W_1 + 0.13W_1 + W_4 + W_3,$$

or

$$0.82\,W = 1.13W_1 + W_4 + W_3 \tag{1}$$

but

$$W_1 = \frac{W - 0.75(0.08W) - 0.5(1800)}{\dfrac{100000 \times 0.16}{12R^{\frac{3}{2}}\,S^{\frac{1}{2}}} + 1} = \frac{0.94W - 900}{\dfrac{1333.3}{R^{\frac{3}{2}}\,S^{\frac{1}{2}}} + 1} \tag{2}$$

Substituting (2) in (1):

$$0.82W = \frac{1.13(0.94W - 900)}{\dfrac{1333.3}{R^{\frac{3}{2}}\,S^{\frac{1}{2}}} + 1} + W_4 + W_3$$

Solving for W

$$W = \frac{\left(1333.3 + R^{\frac{3}{2}}\,S^{\frac{1}{2}}\right)(W_4 + W_3) - 1017\,R^{\frac{3}{2}}\,S^{\frac{1}{2}}}{1093.3 - 0.2422\,R^{\frac{3}{2}}\,S^{\frac{1}{2}}}$$

In plotting the results, W_5 is substituted for $W_4 + W_3$ and curves of constant $\frac{W_5}{P}$ are plotted on coordinates of $\frac{W}{P}$ versus $\frac{W}{S}$ and R.

Confirmation of Validity of Wing Weight Formula

It is desirable to show that the wing weight formula is sufficiently accurate for use in this report. For this purpose it can be used in the form:

$$W_1 = \frac{W - 0.75(W^2) - 0.5(W^3)}{\dfrac{100000 \quad t}{f \; R^{\frac{3}{2}} \; S^{\frac{1}{2}}}}$$

The data from certain actual airplanes are tabulated below:

(1) Air-plane	(2) W	(3) W_2	(4) W_3	(5) f	(6) t	(7) R	(8) S	(9) W_1 (Calculated)	(10) W_1 (Actual)	(11) $\frac{(9)}{(10)}$
P39D	7620	387	979	12	0.15	5.42	213	877	931	0.94
P40F	8523	500	898	12	.15	5.90	236	1152	1128	1.02
P43	7285	775	116	12	.16	5.79	224	902	958	.94
P47D	13220	1116	1806	12	.16	5.61	300	1689	1533	1.10
P51	7967	648	485	12	.14	5.82	236	1111	1070	1.04

The relative error seems as small as can be expected in view of the different types of structures, airfoil shapes, and cutouts.

Maximum Rolling Velocity

In calculating the rate of roll of pursuit airplanes, two cases are considered: (a) airplanes rolling with maximum aileron deflection, (b) airplanes rolling with a 30-pound stick force.

In the first case $\frac{pb}{2V} = K$. For one of the best pursuit airplanes in present production $K = 0.084$. Substituting $p' \times \frac{\pi}{180} = p$, and $V_{mph} = \frac{V}{1.467}$ in order to obtain the units desired the formula as used becomes: $p' = 14.12 \frac{V_{mph}}{b}$.

In the second case it is realized that the rate of roll with constant stick force is critically dependent on the percent aerodynamic balance of the ailerons and that this degree of balance is different for different airplanes, a representative airplane known to have light controls is chosen as a basis for the calculation of the rate of roll at constant stick force. The rolling velocity can be stated as $p' = \frac{K}{\rho b^4 V}$ for similar airplanes. From the performance of the airplane used as a standard, K is calculated to be $K = 130,200,000$. Substituting this value in the above equation and changing units we have $p' = \frac{88740000}{\rho b^4 \, V_{mph}}$.

APPENDIX C

DISCUSSION OF THE LIMITATIONS AND COORDINATES OF CHARTS

Take-off Run

The take-off run is largely dependent on the handling of the controls of the airplane, and varies considerably for different pilots. By assuming a systematic handling of the controls we get a reasonable distance required for take-off and the curves show the trends to be expected when all the airplanes take off at a lift coefficient of 1.0.

Rate of Climb

There are several factors leading to the selection of climbing at the attitude for maximum L/D. If each airplane had to be investigated as to the horsepower available and the horsepower required at several speeds, the work would be unnecessarily involved in finding the maximum rates of climb. Consequently some systematic variation of climbing speeds is desirable. For all the practical airplanes on the charts, the rate of climb at maximum L/D is approximately equal to the maximum possible rate of climb.

The assumption of relatively high propeller efficiencies in the climb condition is justified since most of the forward speeds while flying at maximum L/D are high.

High Speeds

The actual values of speed are dependent upon the assumptions of drag, aspect ratio, propeller efficiency, and altitude but the curves should prove useful in showing the performance obtainable in new designs, and in showing the gains in speed that can be obtained in existing designs by changes in weight, power, wing area, or altitude.

Minimum Radius of Turn

The conditions for steady turning are that the airplanes shall not stall, change their altitudes, speeds, or require more power than that available. For most of the light-weight airplanes, particularly at low altitudes, the minimum radius of steady turn occurs close to the stall. Wherever needed it is assumed that flaps can be used with optimum deflections. The maximum lift coefficient with flaps deflected fully is taken as 2.5.

The minimum radius of accelerated turn occurs in a vertical bank and momentarily the pilot can lose altitude or use up part of the kinetic energy of the airplane to increase effectively the power available for the maneuver. This makes the radius of turn a function of only $C_{L_{max}}$, ρ, and W/S, so the steady turn charts can be used to give the minimum radius of accelerated turn by using the correct wing loading for the airplanes but a power loading of zero.

Minimum Time to Turn

The same assumptions used for the minimum radius of turn calculations also apply to the calculations for minimum time to turn. As a whole the speeds involved are slightly higher, propeller efficiencies are higher, and lift coefficients are slightly lower at the speeds for minimum time to turn.

The Effect of Aspect Ratio

A particular airplane (airplane A: W/P = 6, W/S = 40, R = 5.75) is chosen for further investigation of the effect of changing the aspect ratio on all the performance. This method of investigating aspect ratios must not be considered sufficient for final selection of the desired aspect ratio. For instance, the airplane considered above may be the most desirable airplane for a specific mission at 25,000 feet, and by varying the aspect ratio, keeping wing area constant, we may be able to improve the performance; however, the original values of power loading and wing loading do not apply to the new airplane since the structural weight has changed. Figures 9 to 14 allow for the variation in structural weight, and these show the performance to be expected.

The Effect of Wing Loading

Airplane A is again chosen to show what happens to the performance of an airplane when the wing loading is varied by changing the wing area, allowing for the change in structural weights, keeping the aspect ratio equal to 5.75. Figure 7 is extremely important in that it may modify considerably the conclusions drawn from the other figures. Its use is also important because it gives a rapid method of estimating the available wing and power loadings of an airplane having a certain ratio of internal weights to horsepower.

Coordinate System

The only coordinate system requiring an explanation is one using power loading as the ordinate and wing loading as the abscissa. These generalized coordinates allow the superposition of the charts to form a composite chart to be used for airplanes of all different powers and sizes. The performance estimated from the charts checks flight tests of actual airplanes very well except for the rate-of-roll curves; many airplanes do not have sufficient aerodynamic balance to allow the desired aileron deflection at high speeds to attain the rolling velocities shown. The rate of roll is dependent on the size of the airplanes rather than on power and wing loading and is not included on the latter coordinates because the chart would have to be limited to one definite power.

The foregoing discussion gives the uses and limitations of the charts. The use of the charts is the presentation of airplane selection in a manner which shows the actual compromises being made. Airplane designers will probably have other assumptions they would prefer for building charts of their own; they can use the included formulas to adapt the charts to their use, or can use weight variation curves of their own with the given selection charts provided the aspect ratio of the airplanes under consideration is essentially the same as that used in the charts.

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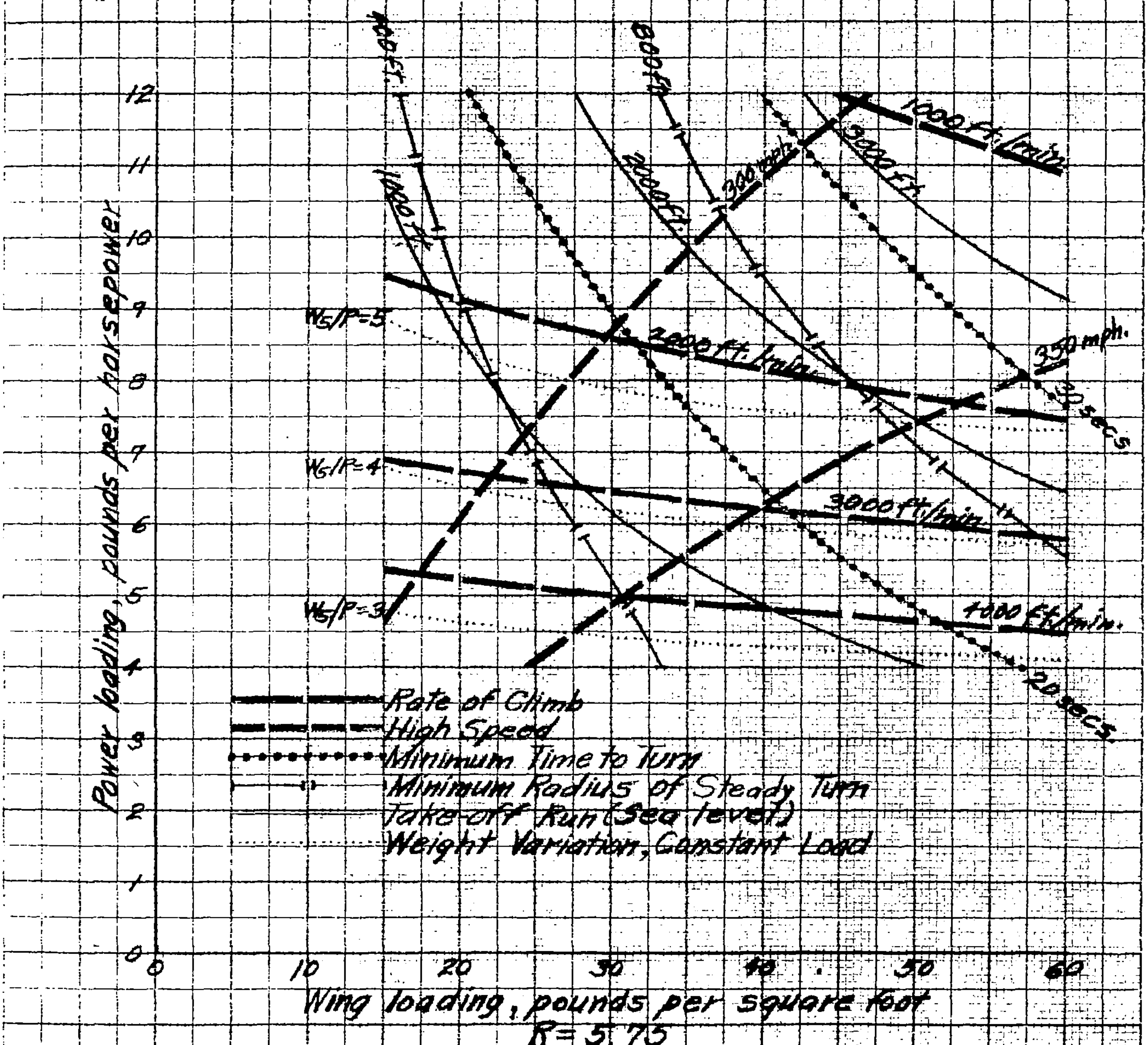

FIGURE 1(a) - COMPOSITE SELECTION CHART - SEA LEVEL

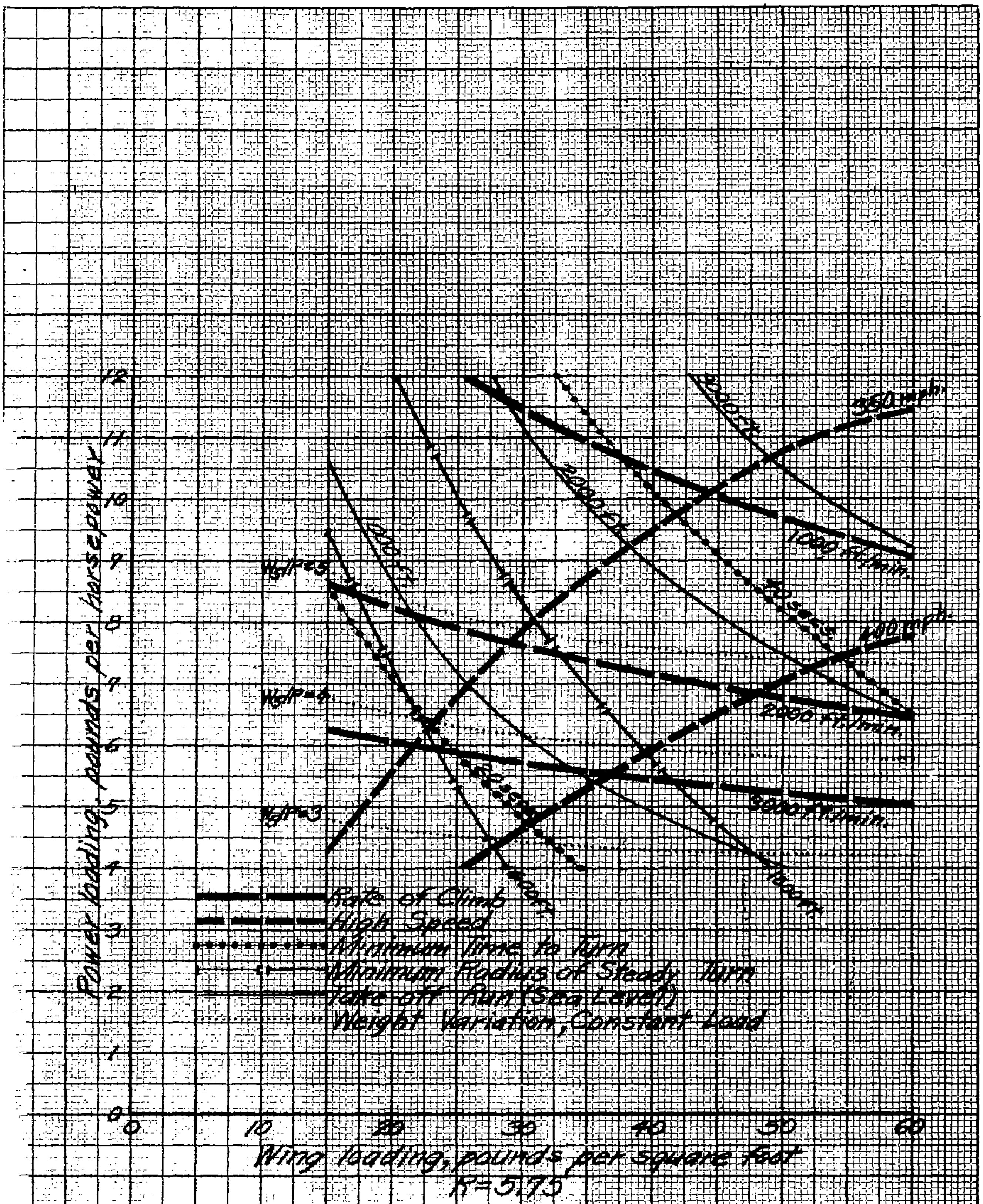

FIGURE 1(b) - COMPOSITE SELECTION CHART - 15,000 FEET ALTITUDE

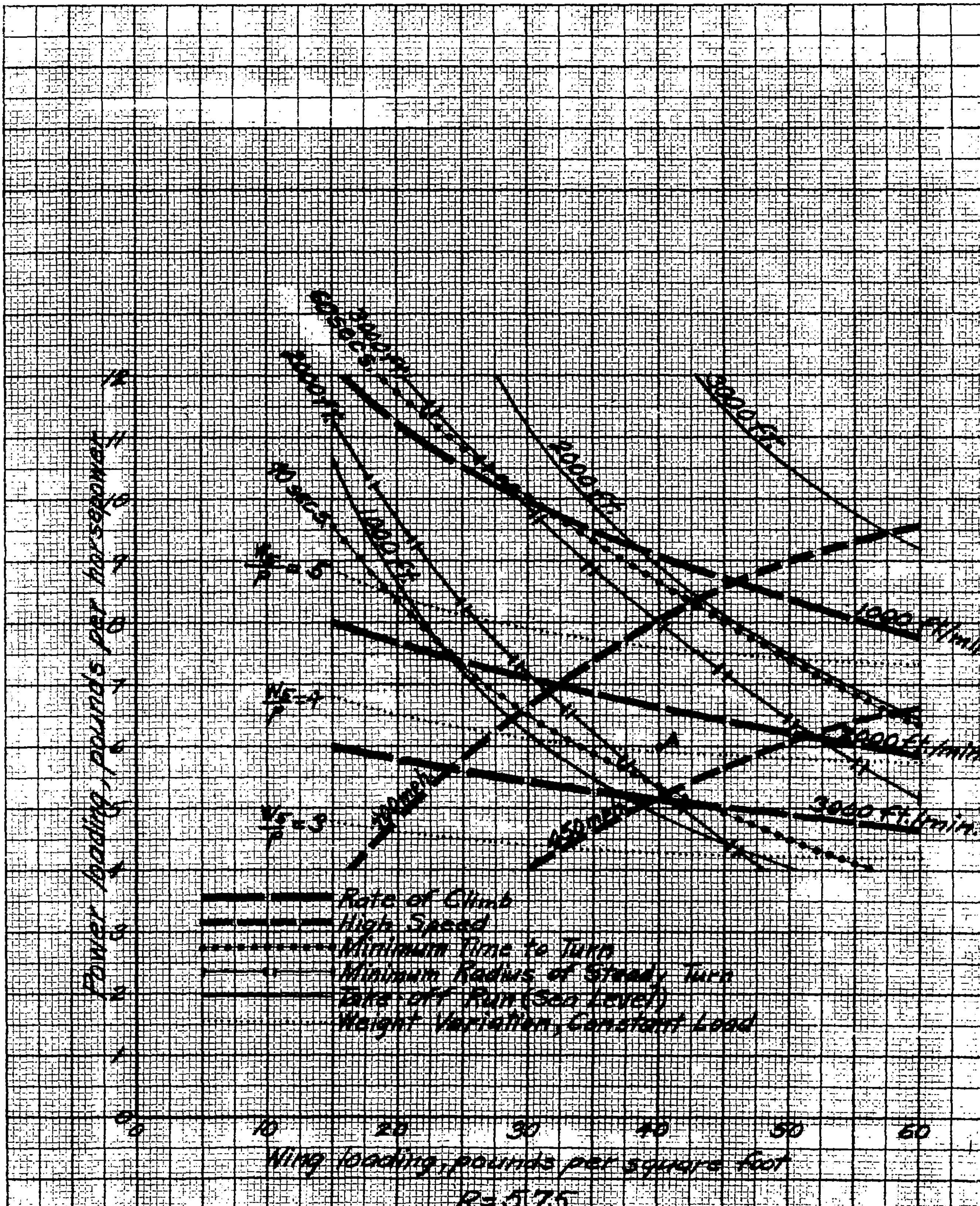

FIGURE 1(C) - COMPOSITE SELECTION CHART - 25,000 FEET ALTITUDE

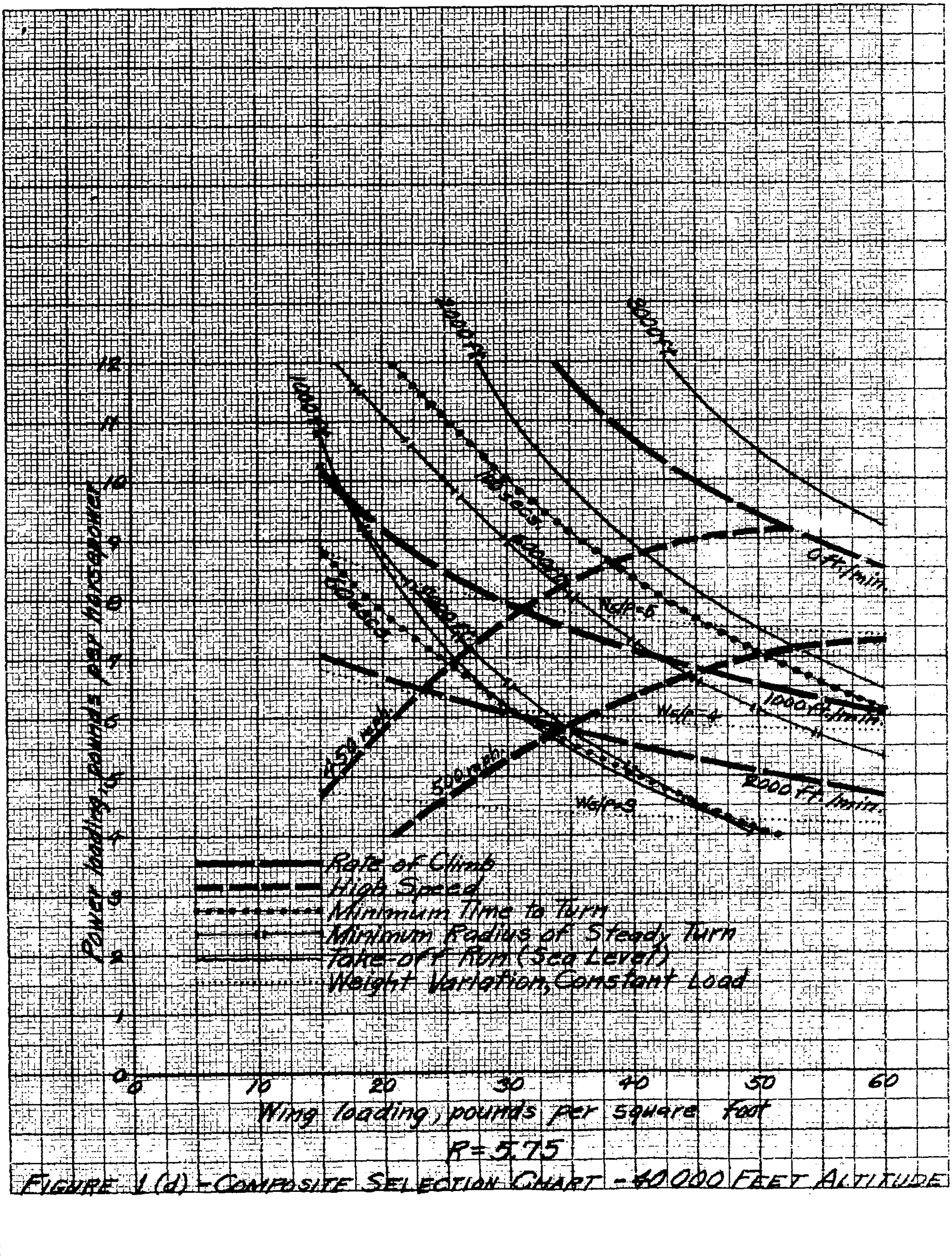

FIGURE 1 (d) - COMPOSITE SELECTION CHART - 40 000 FEET ALTITUDE

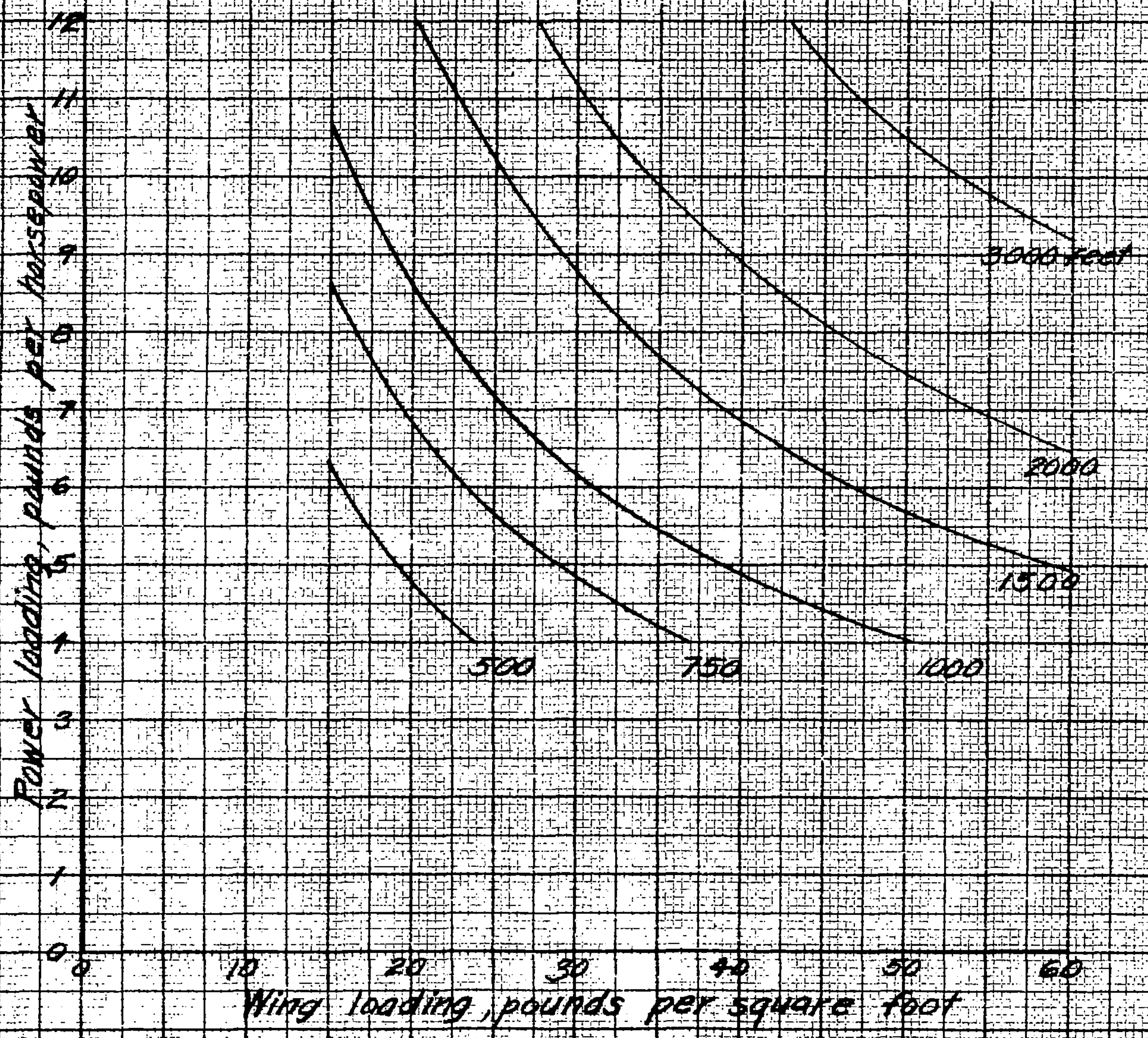

FIGURE 2(a) TAKE-OFF RUN AT SEA LEVEL

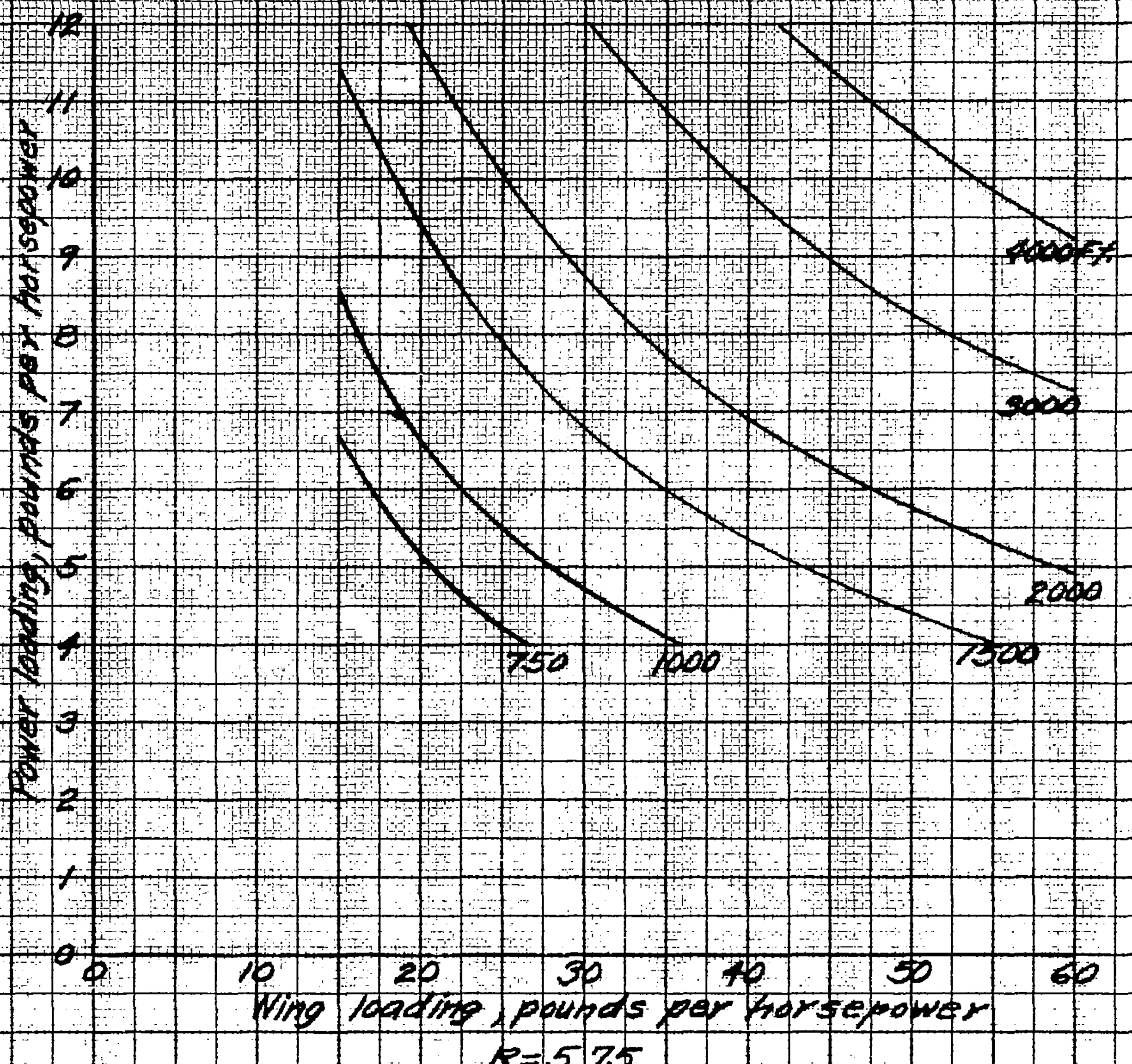

FIGURE 2(b) - TAKE-OFF RUN AT 5000 FEET ALTITUDE

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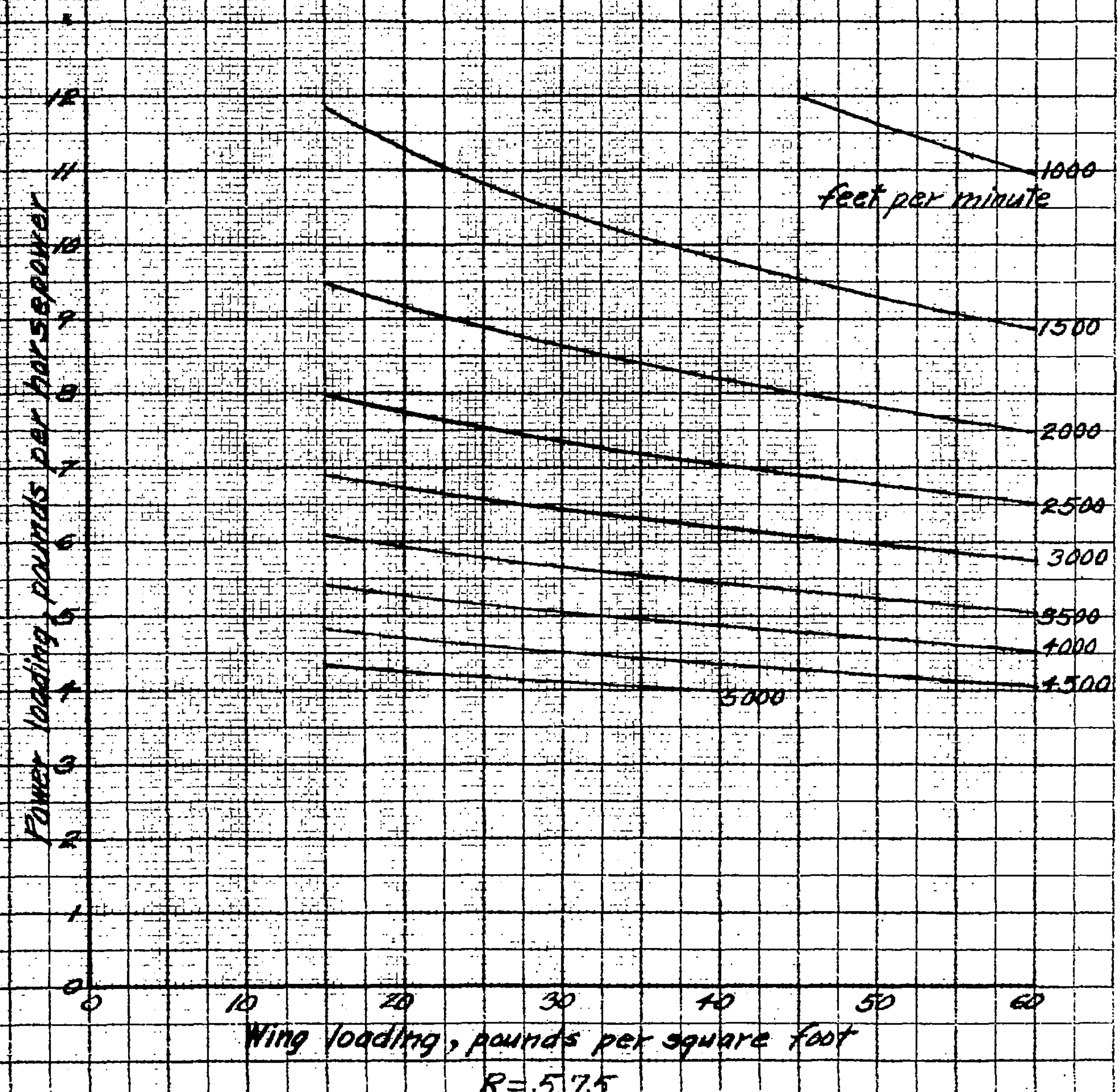


FIGURE 3(a) - RATE OF CLIMB AT SEA LEVEL

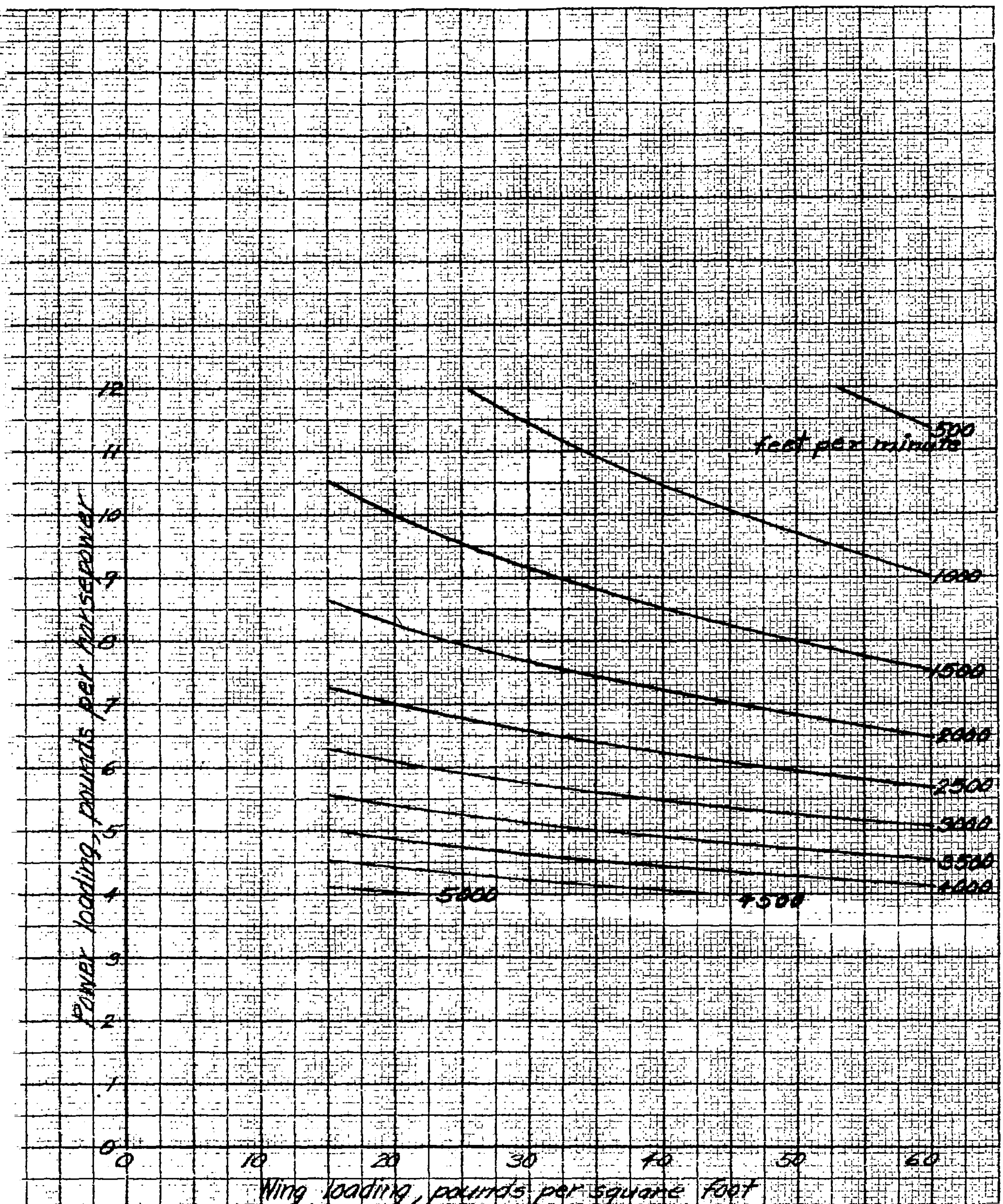
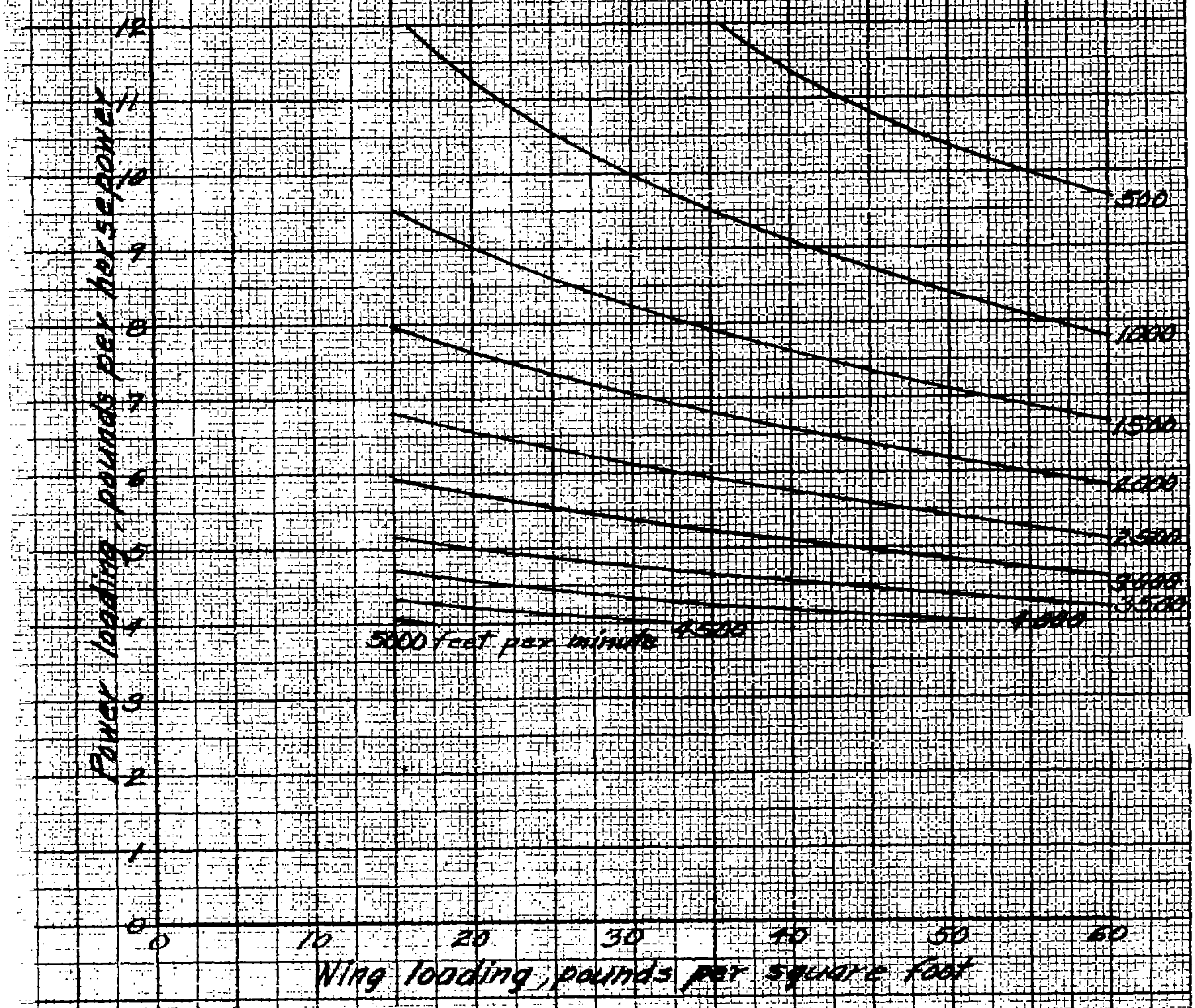


FIGURE 3(b) - RATE OF CLIMB AT 15000 FEET ALTITUDE



R = 5.75

FIGURE 3(c) - RATE OF CLIMB AT 25000 FEET ALTITUDE

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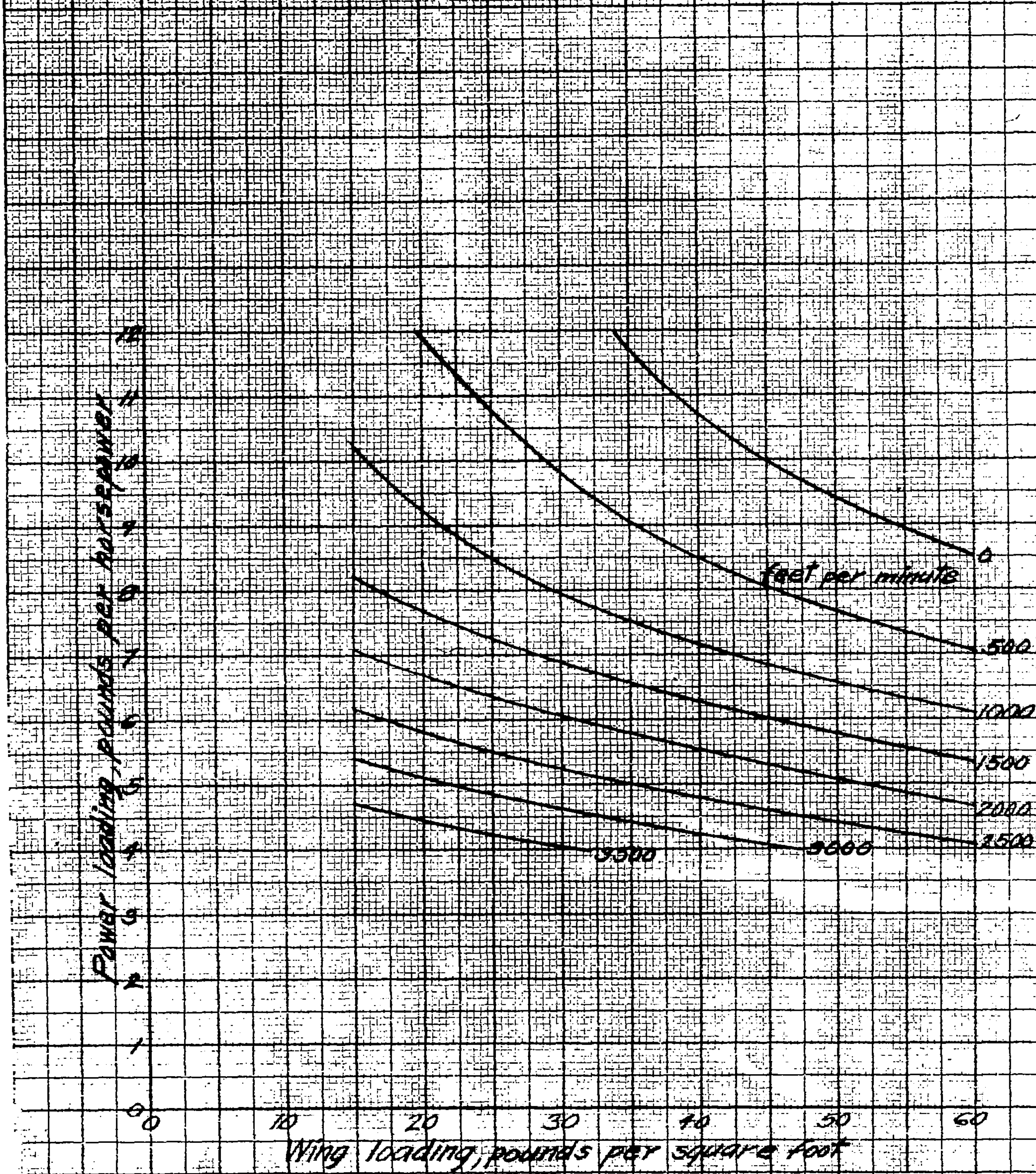

FIGURE 3(d)- RATE OF CLIMB AT 40,000 FEET ALTITUDE

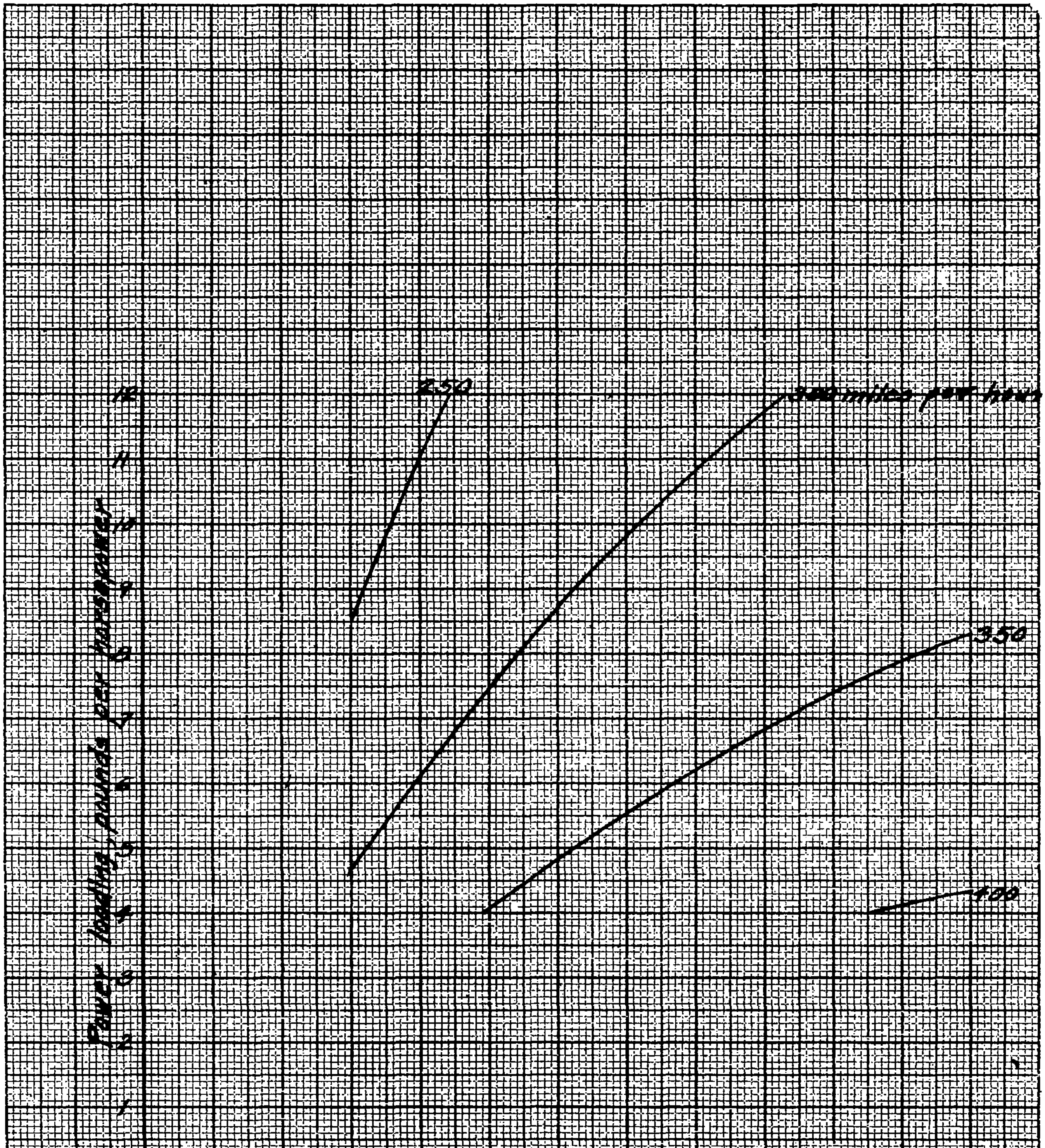

Figure 4(a) – High Speed at Sea Level

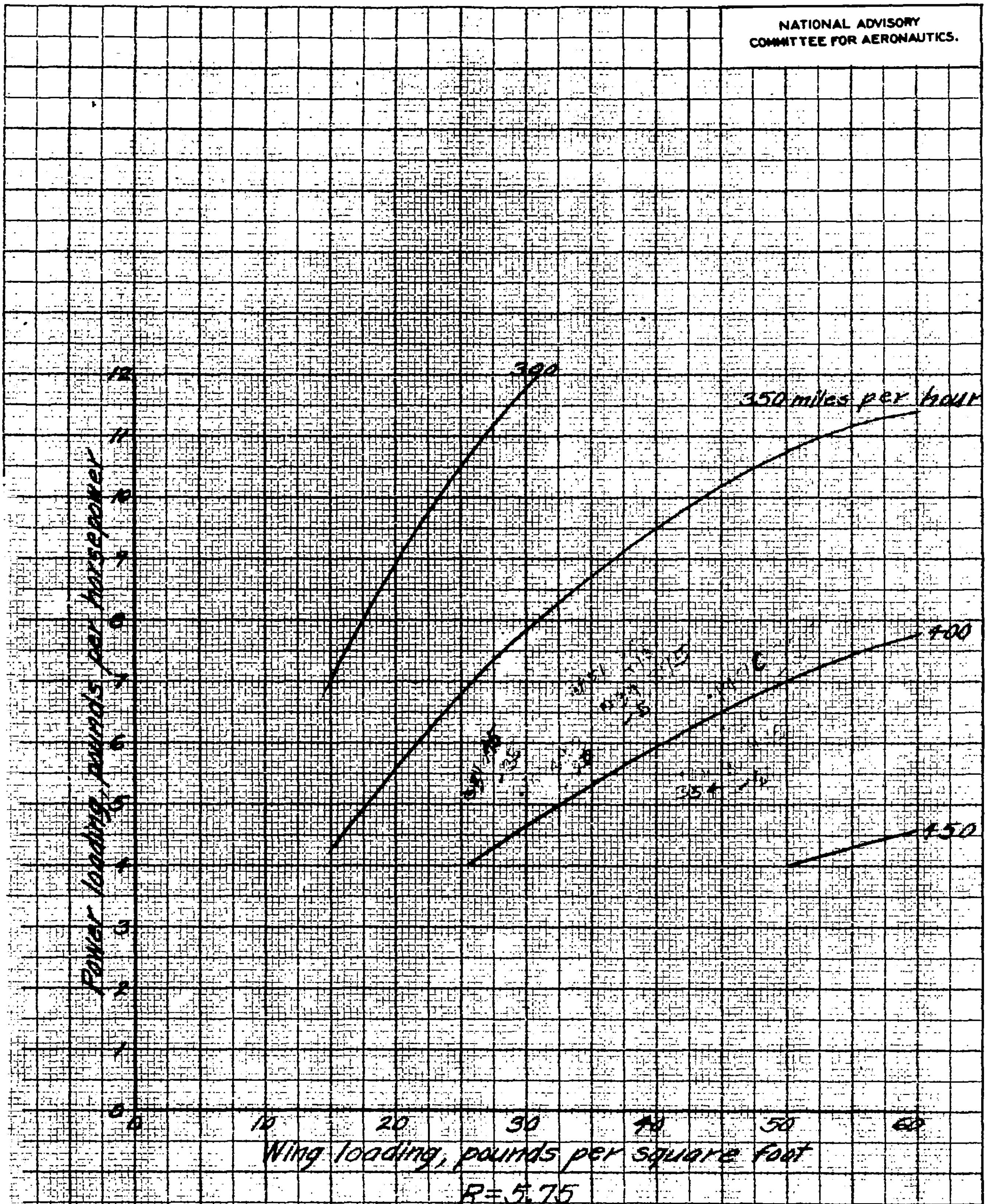

FIGURE 4(b) - HIGH SPEED AT 15000 FEET Altitude

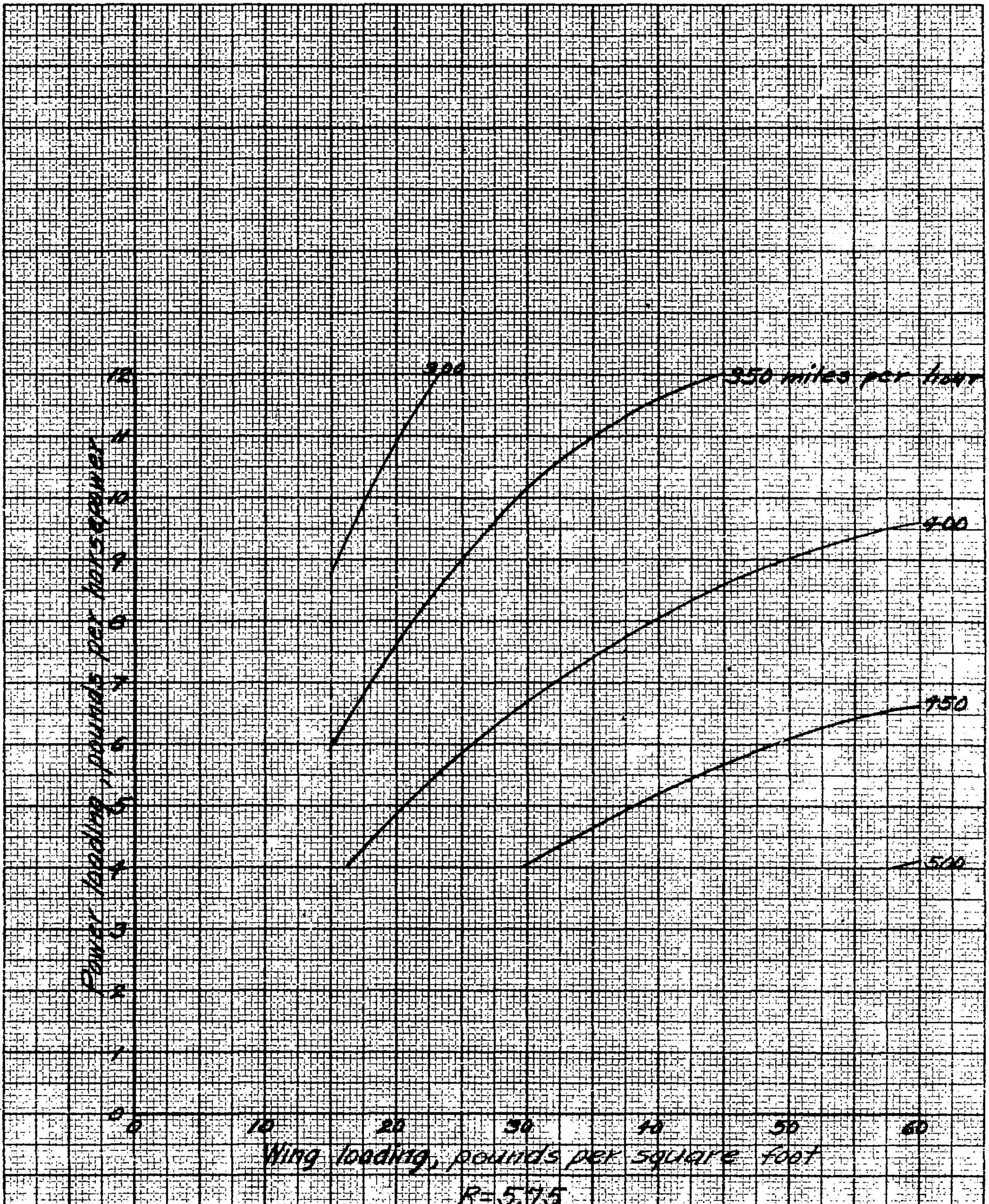

FIGURE 4(C) - HIGH SPEED AT 25 000 FEET ALTITUDE

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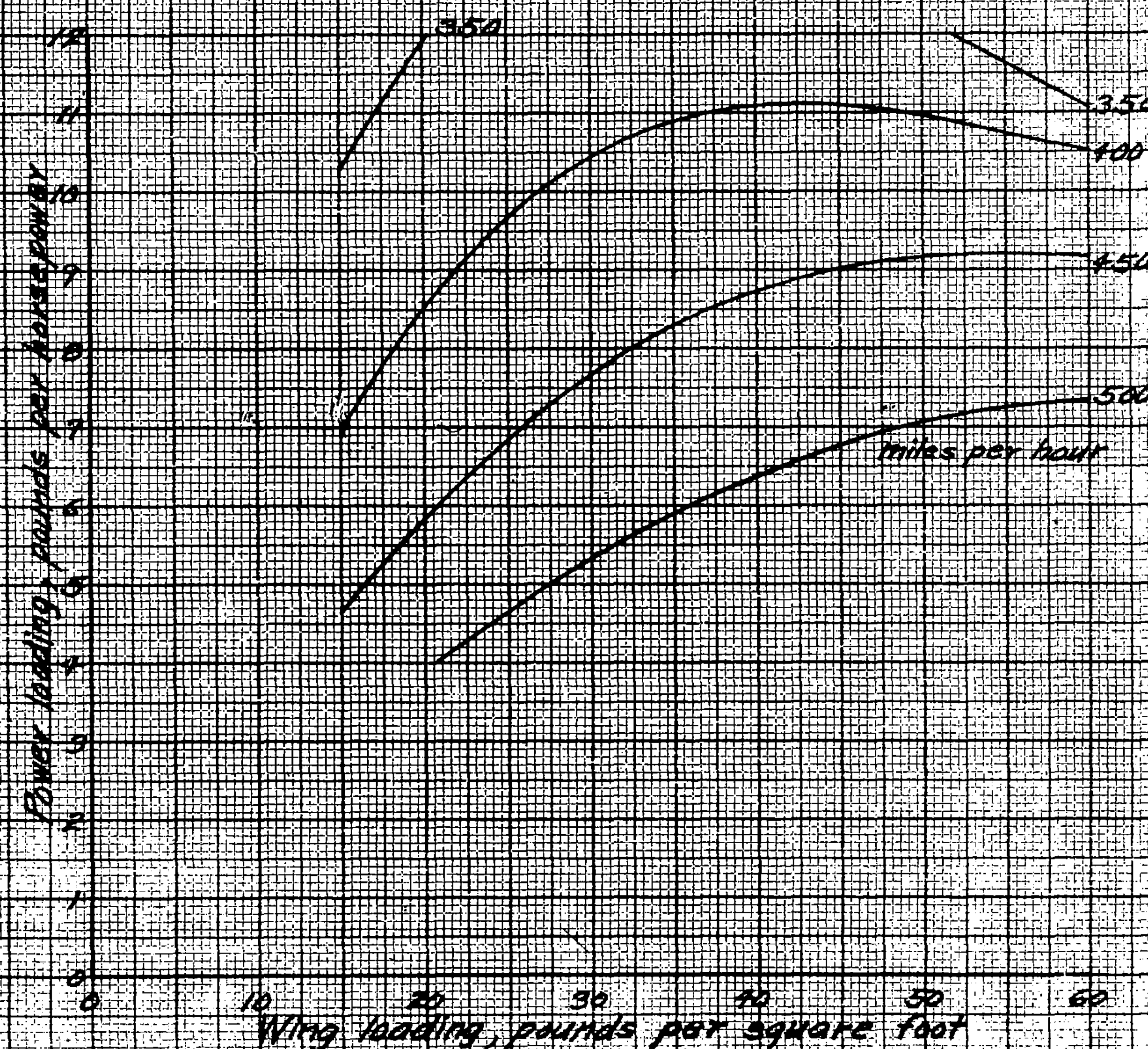

R = 5.75

FIGURE 4(d) - HIGH SPEED AT 40,000 FEET ALTITUDE

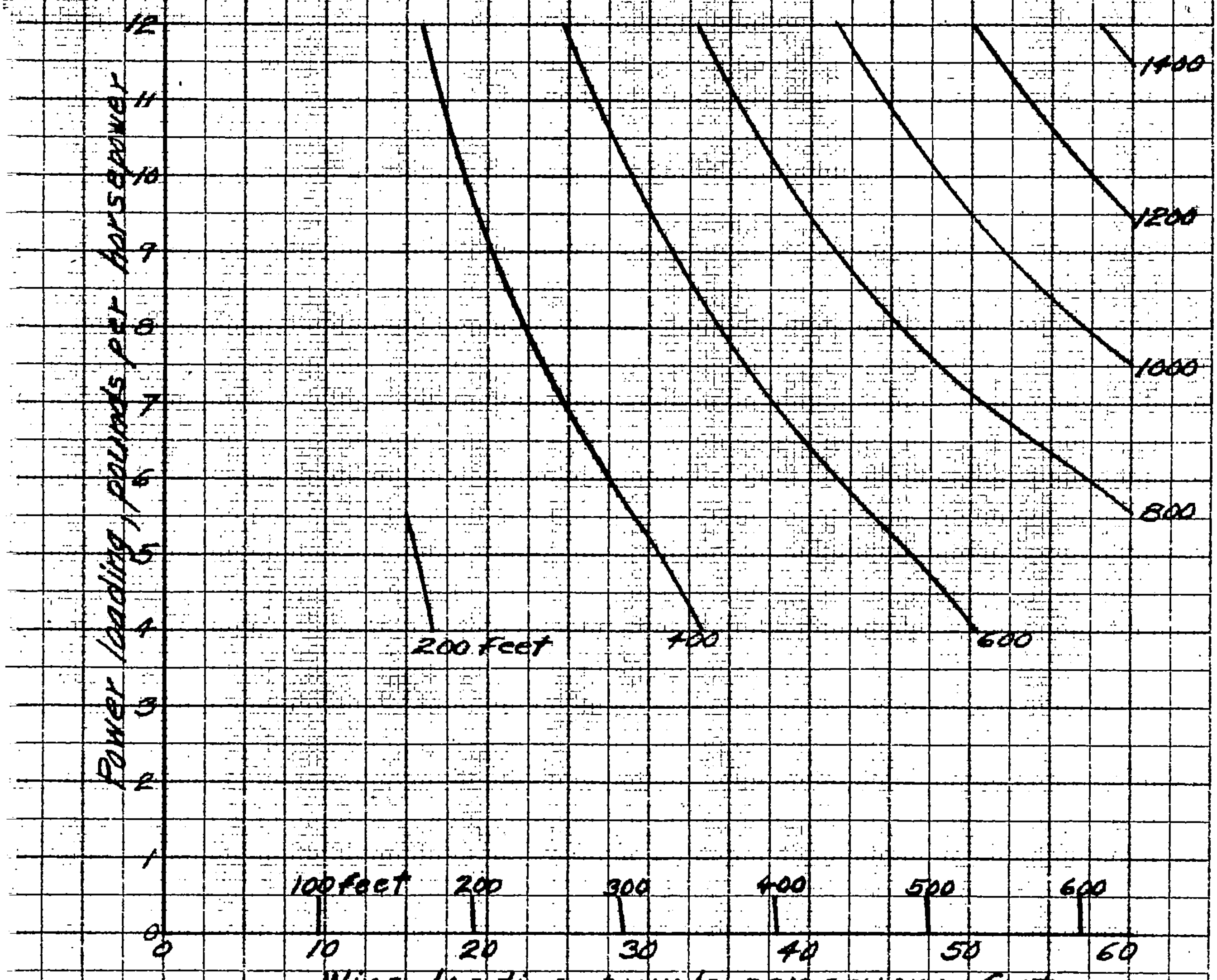

FIGURE 5(a) - RADIUS OF TURN AT SEA LEVEL

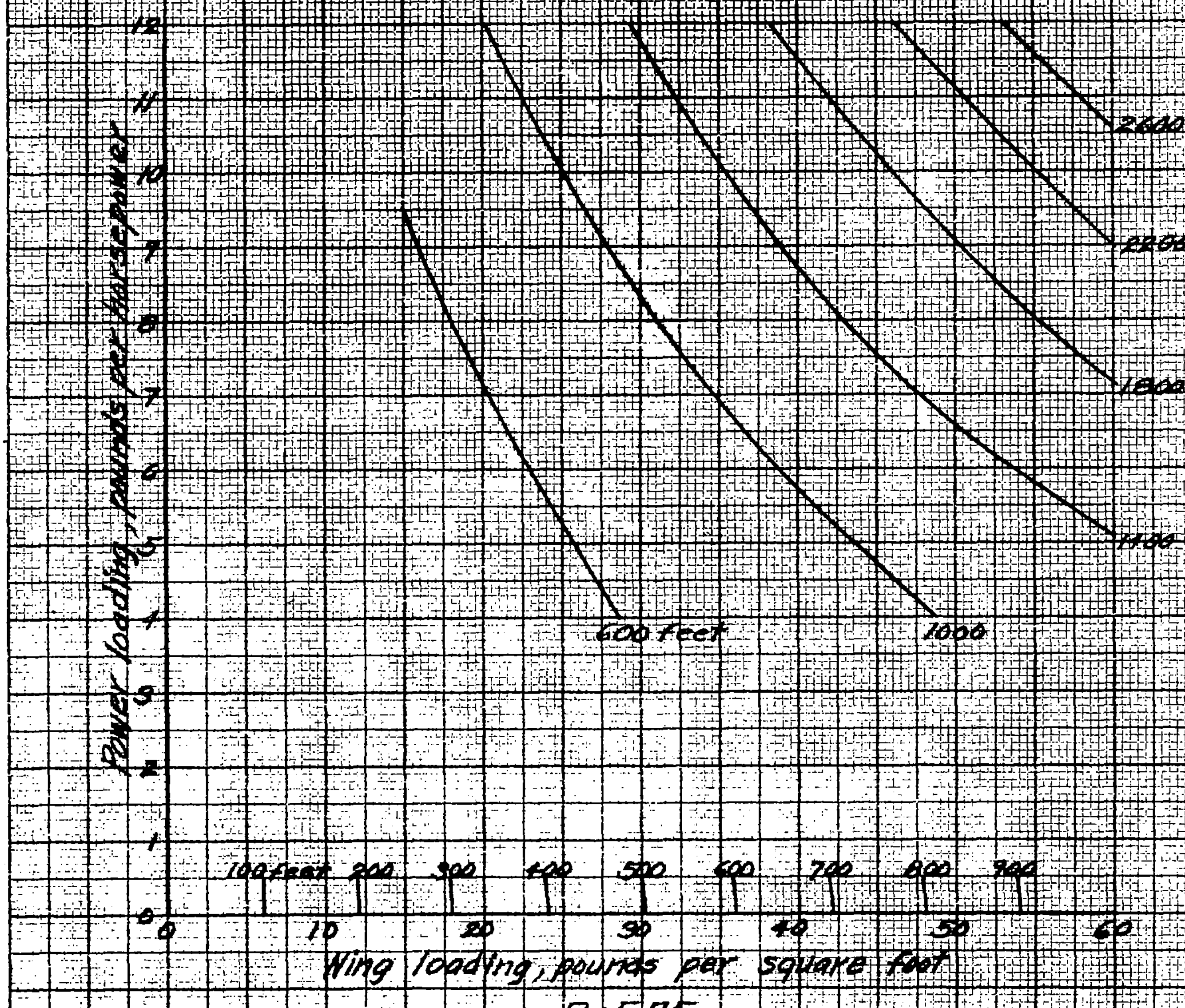

FIGURE 5(b) - RADIUS OF TURN AT 15000 FEET ALTITUDE

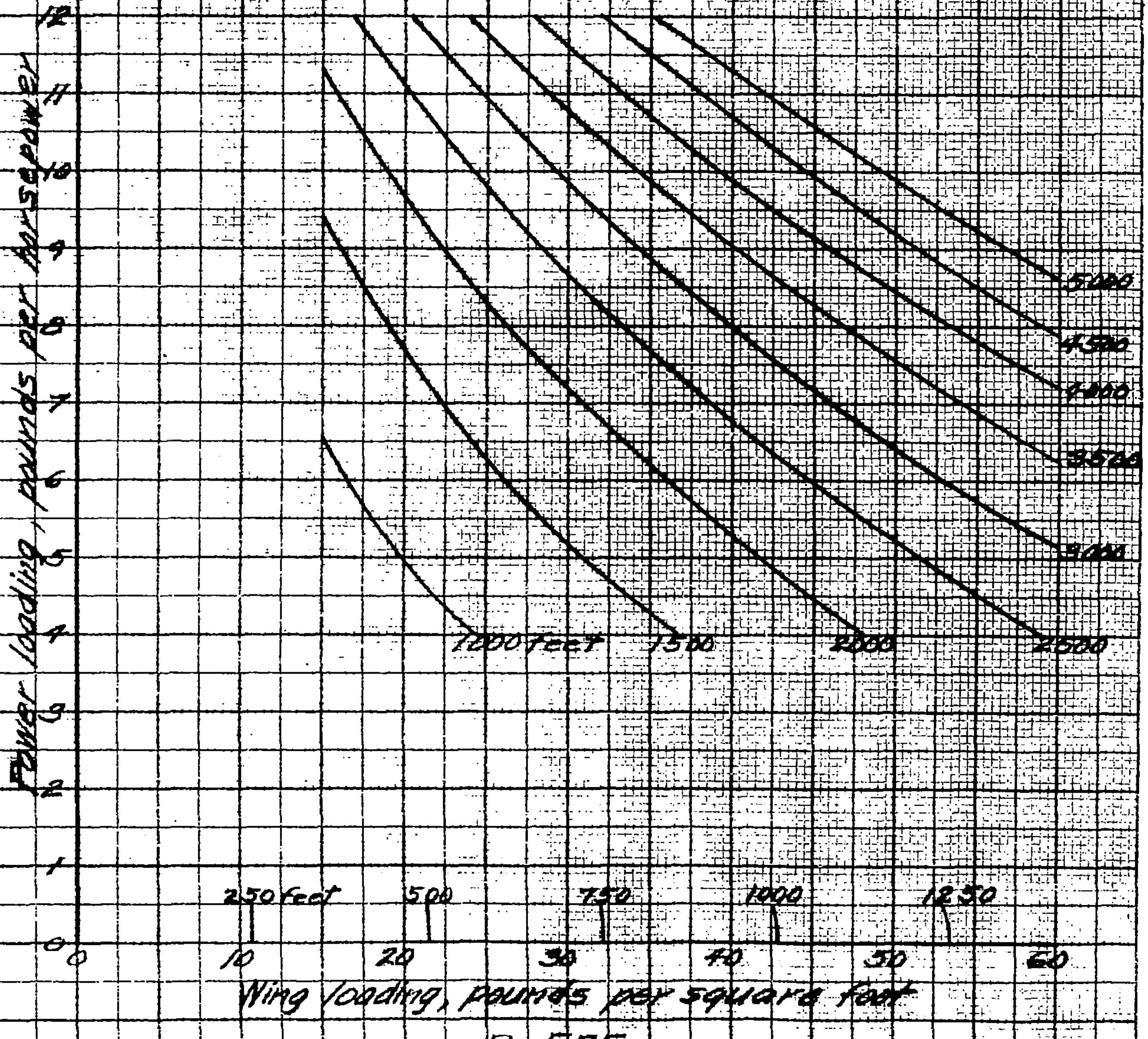

FIGURE 5(c) - RADIUS OF TURN AT 25000 FEET ALTITUDE

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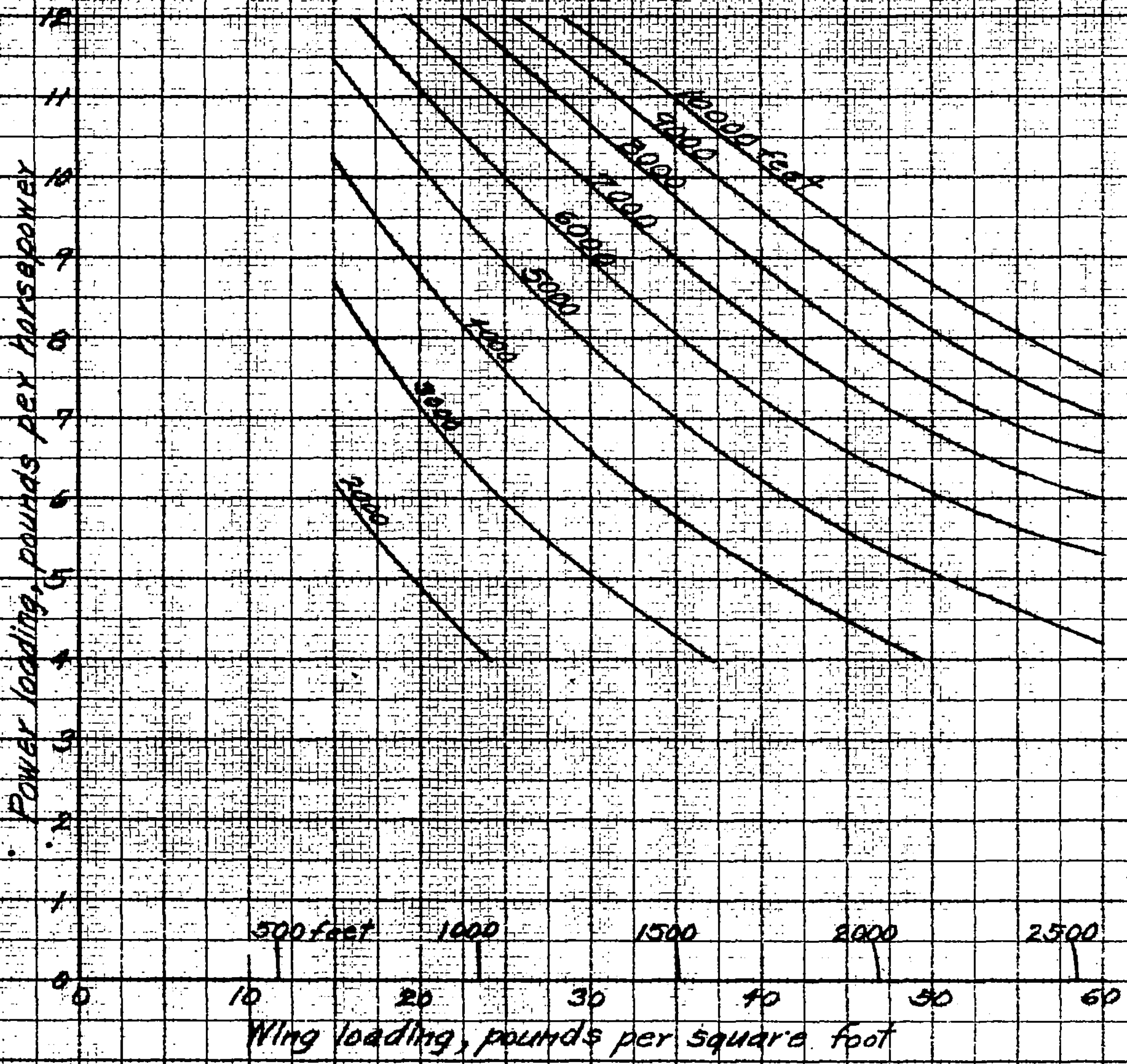


R = 5.75

FIGURE 5(d) – RADIUS OF TURN AT 40000 FEET ALTITUDE

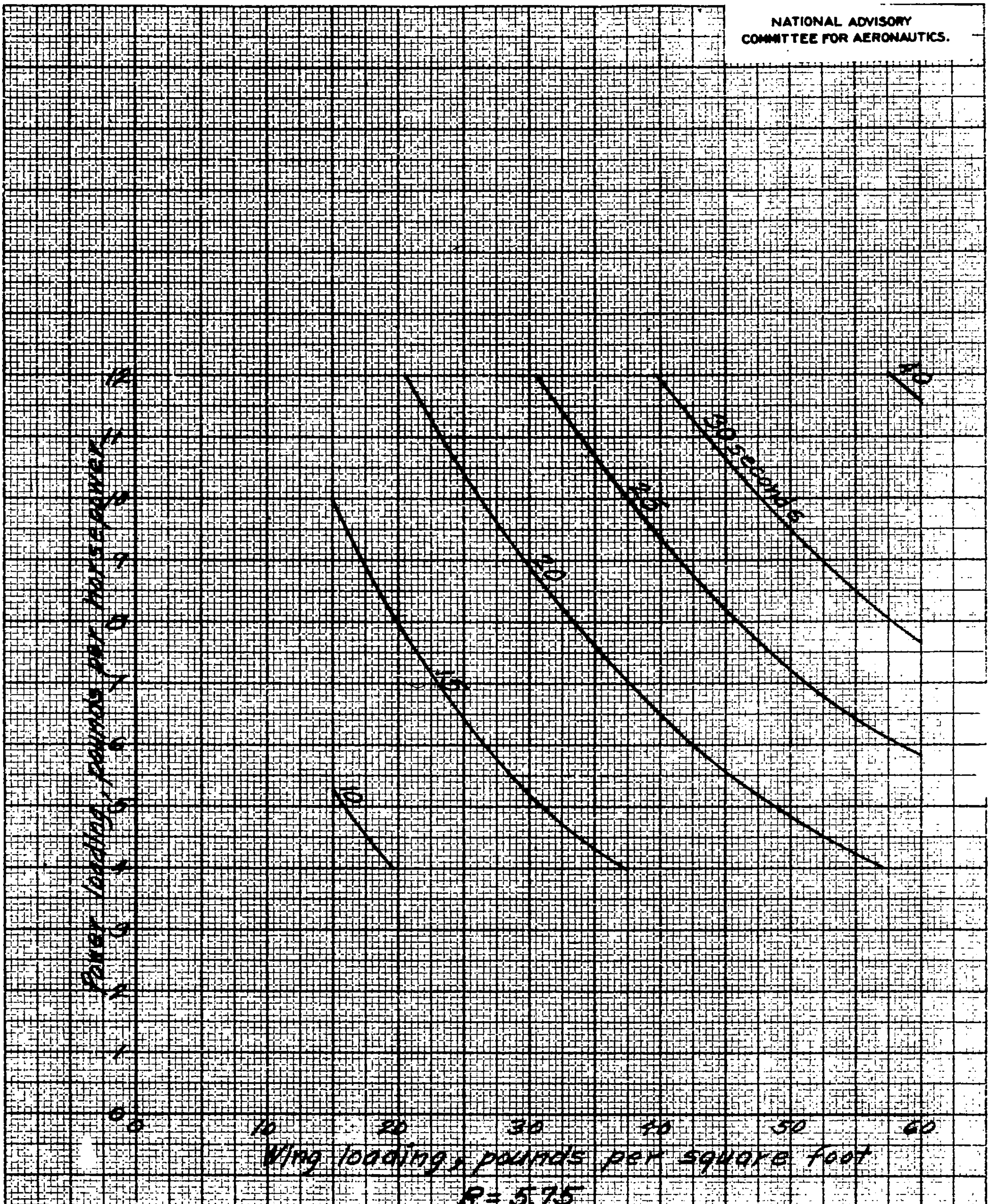
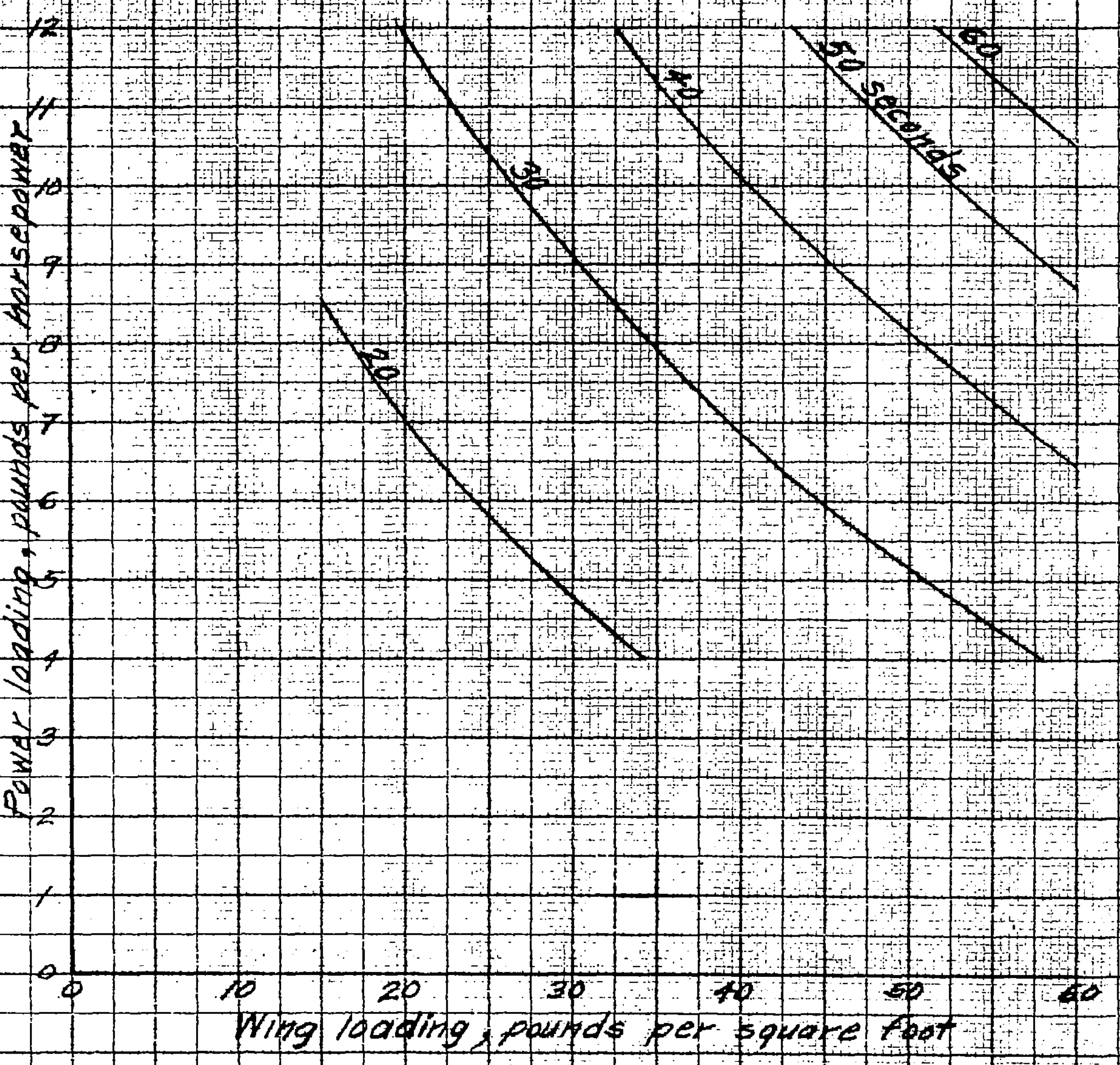
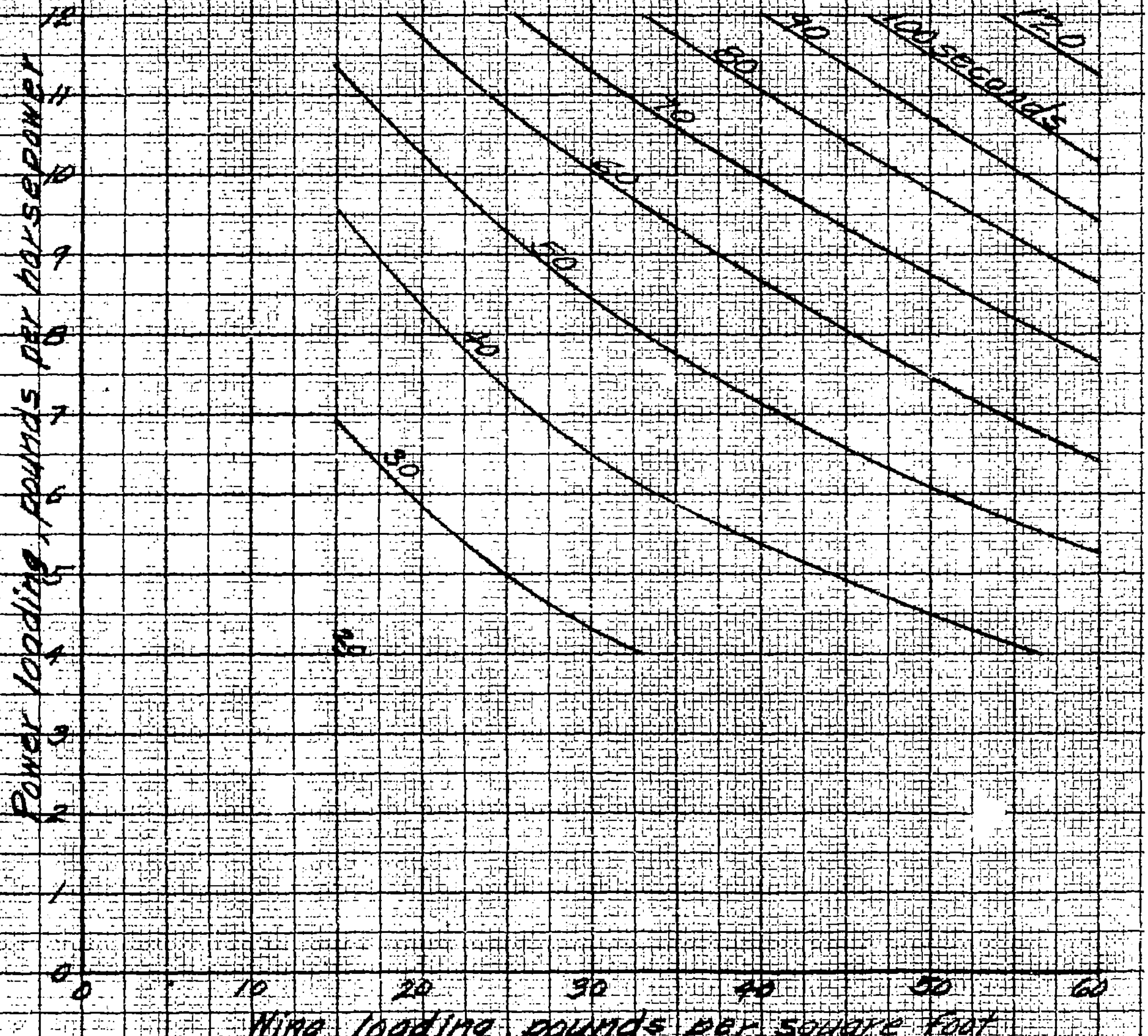


FIGURE 6(a)- TIME TO TURN AT SEA LEVEL



$R = 5.75$

FIGURE 6(b) - TIME TO TURN AT 15000 FEET ALTITUDE



R=5.75

FIGURE 6(c) - TIME TO TURN AT 25000 FEET ALTITUDE

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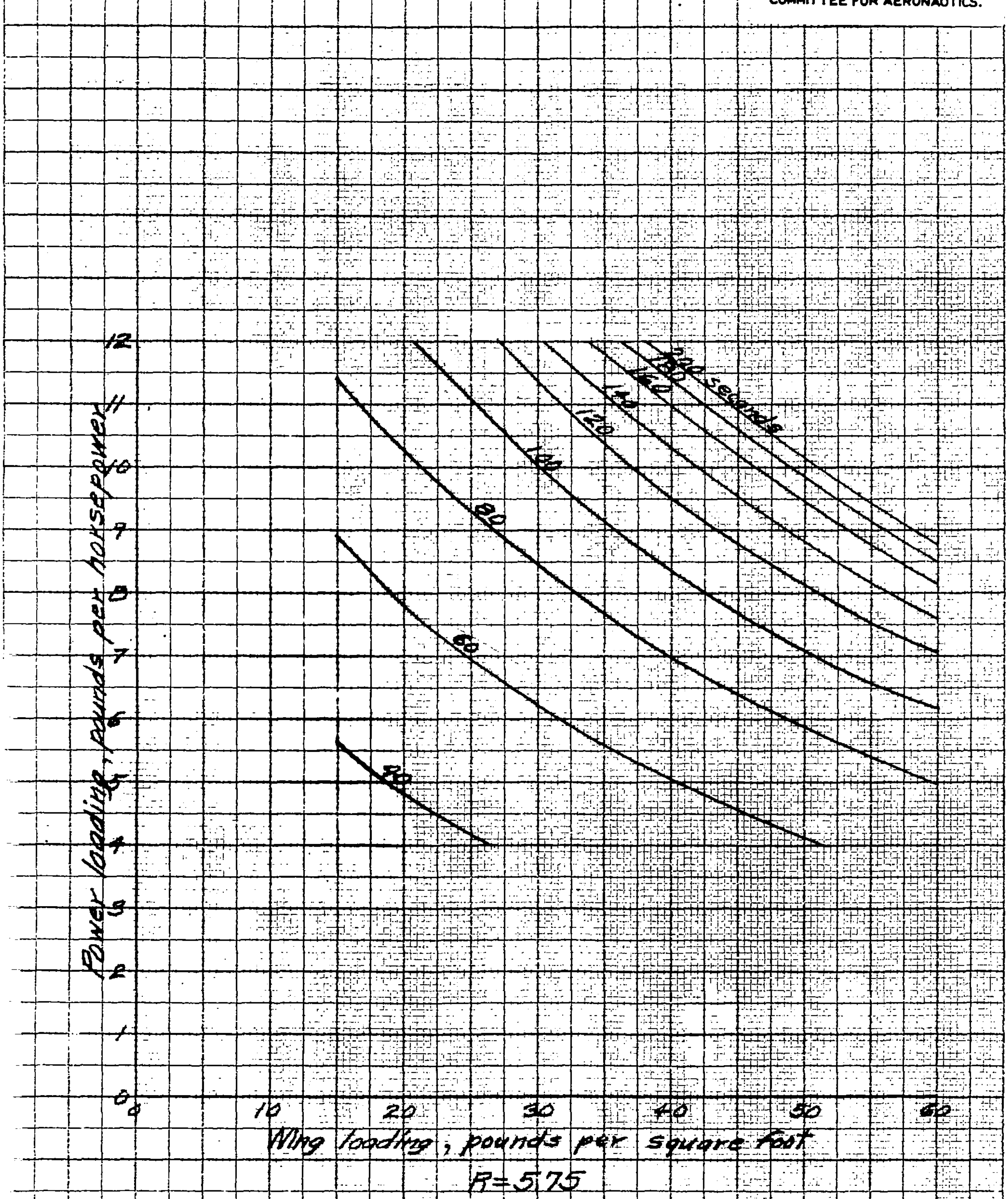

FIGURE 6(d) - TIME TO TURN AT 40,000 FEET ALTITUDE

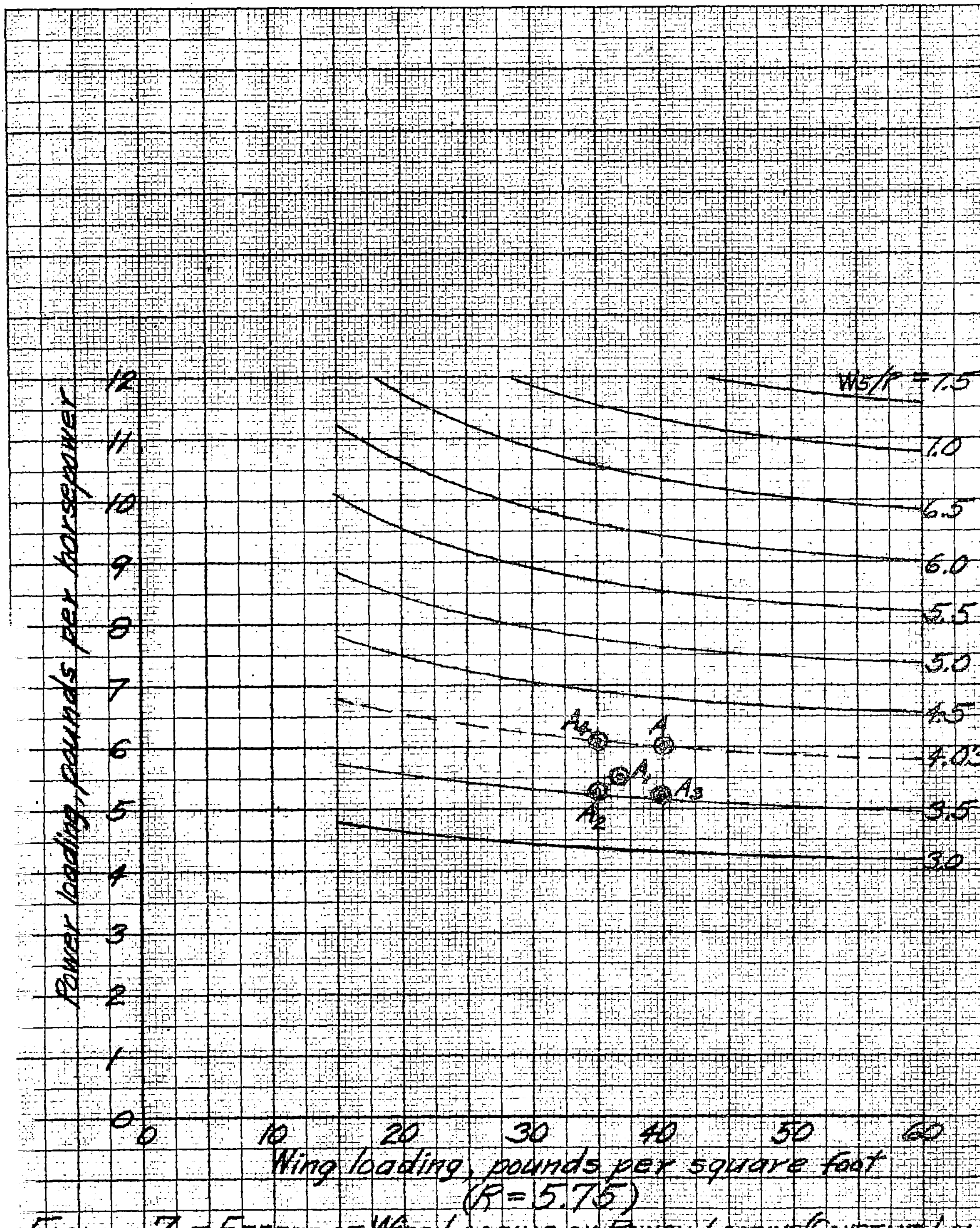

FIGURE 7 – EFFECT OF Wing LOADING ON POWER LOADING (CONSTANT LOADS)

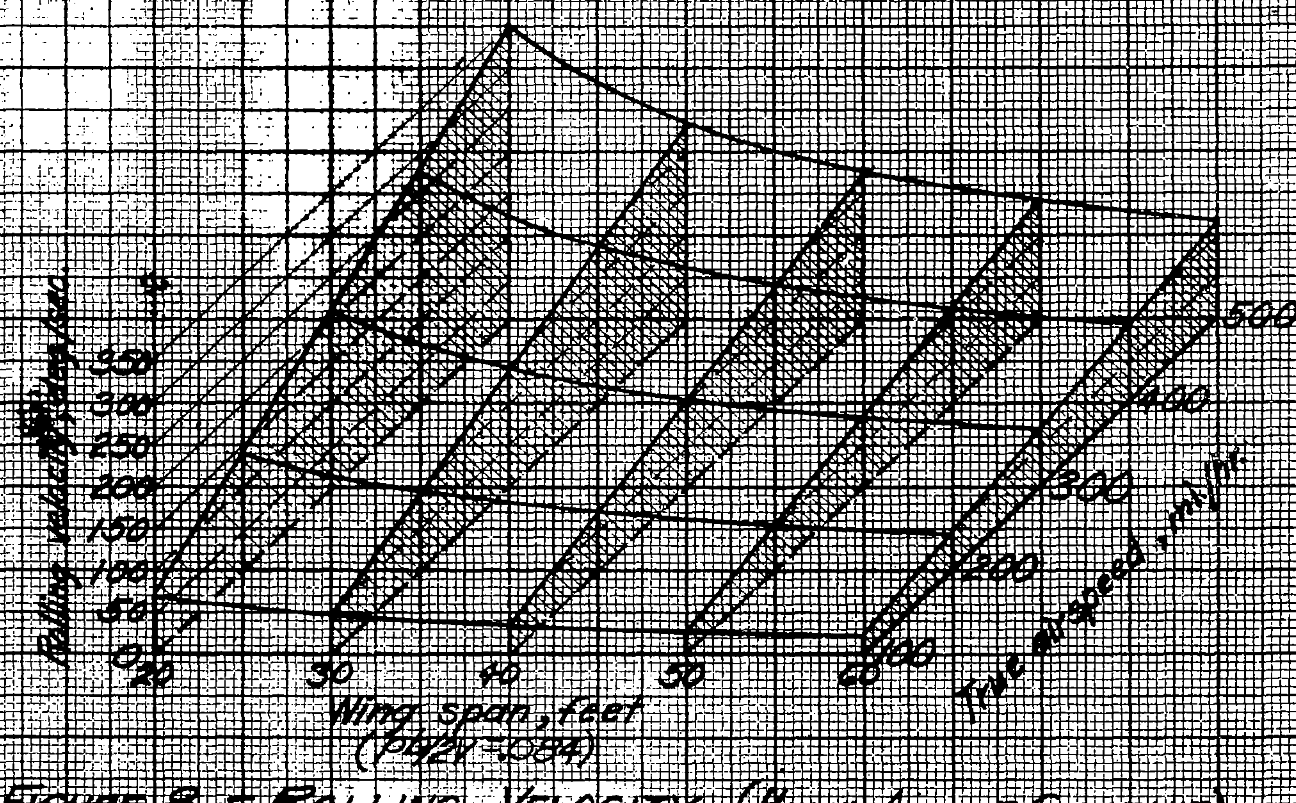

FIGURE 8 - ROLLING VELOCITY (HELIX ANGLE = CONSTANT)

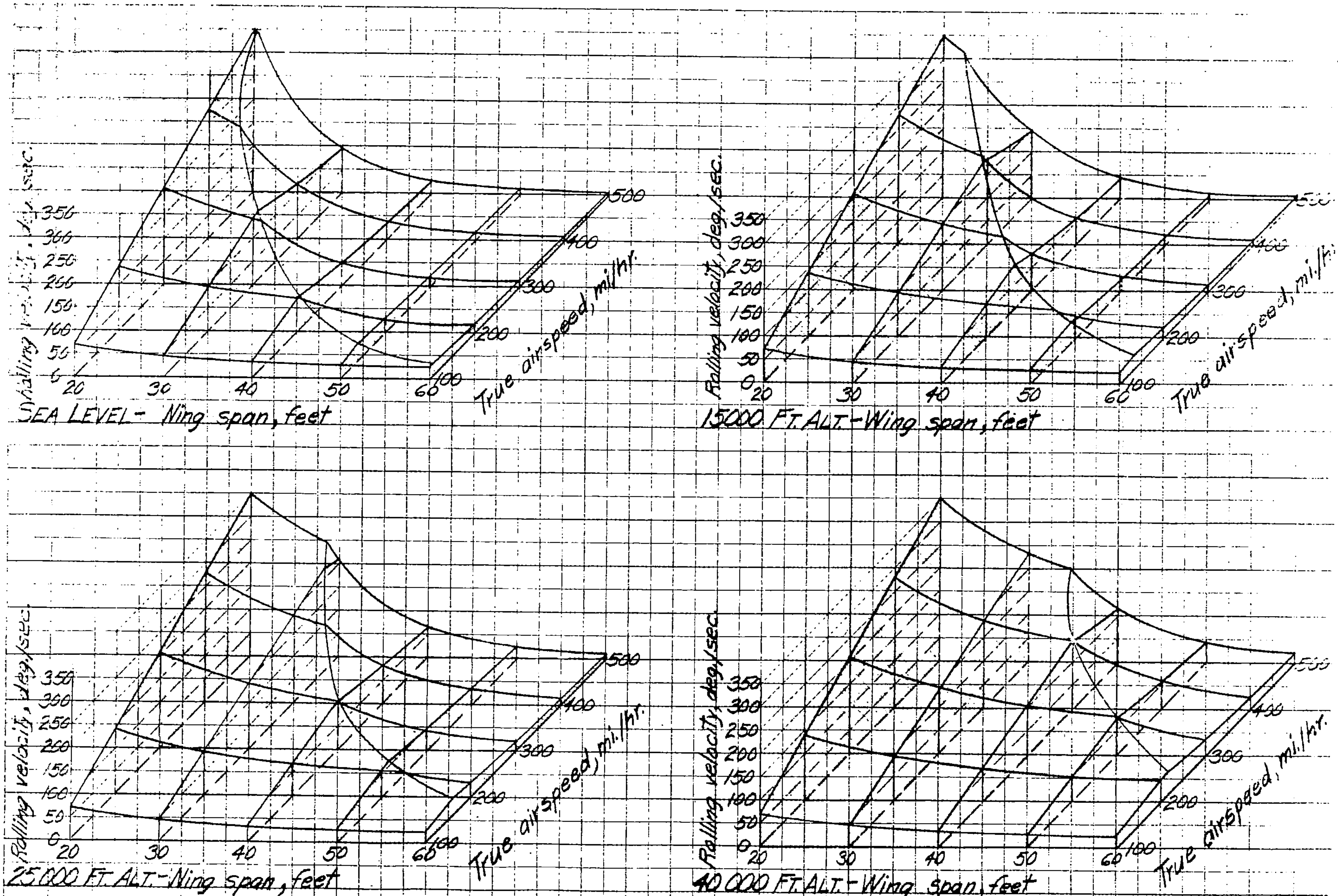

FIGURE 9 - ROLLING VELOCITY WITH 30 POUND STICK FORCE $(R = 5.75, [pb/2V]_{max.} = .084)$

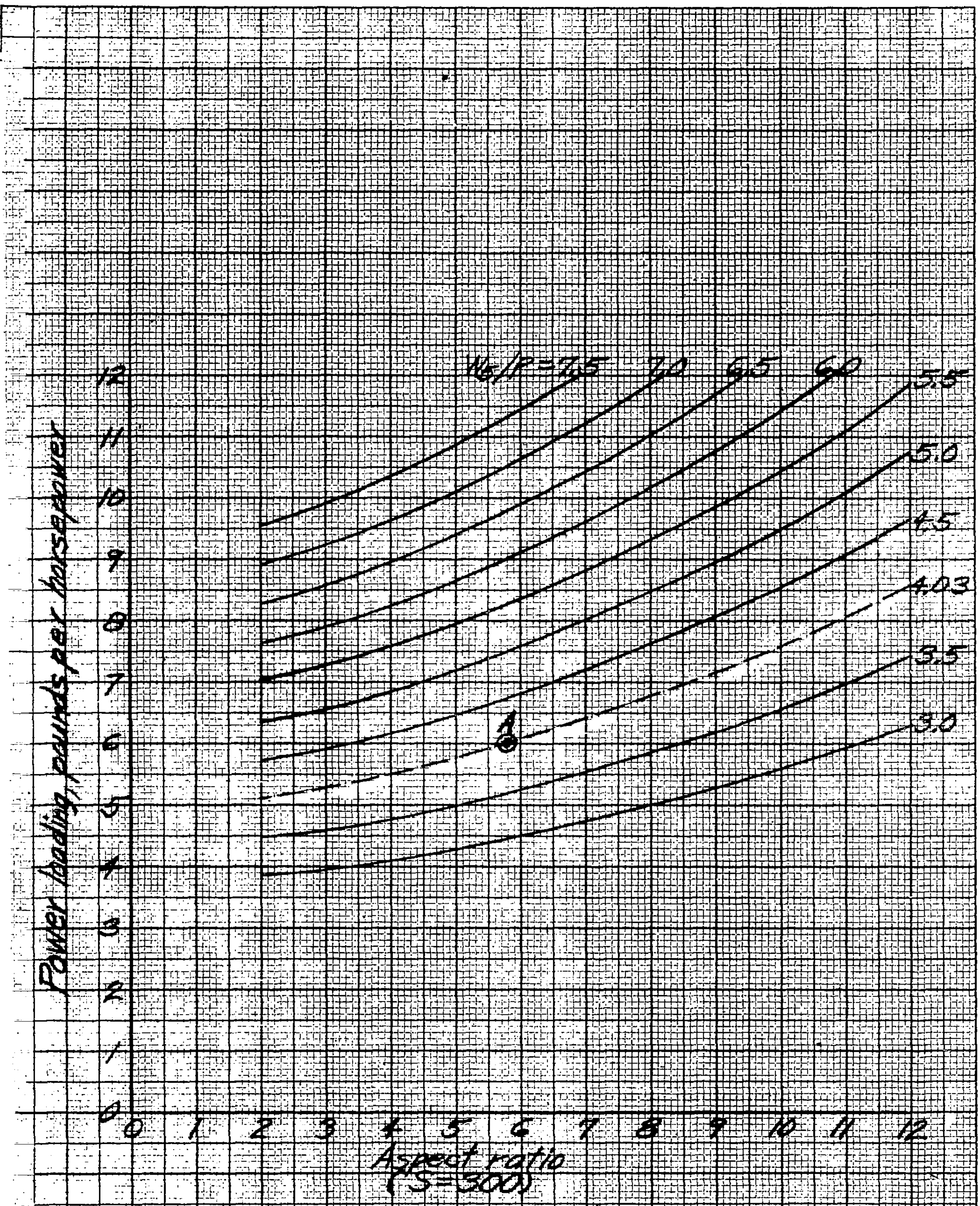

FIGURE 10(a) - EFFECT OF ASPECT RATIO ON POWER LOADING (CONSTANT LOADS)

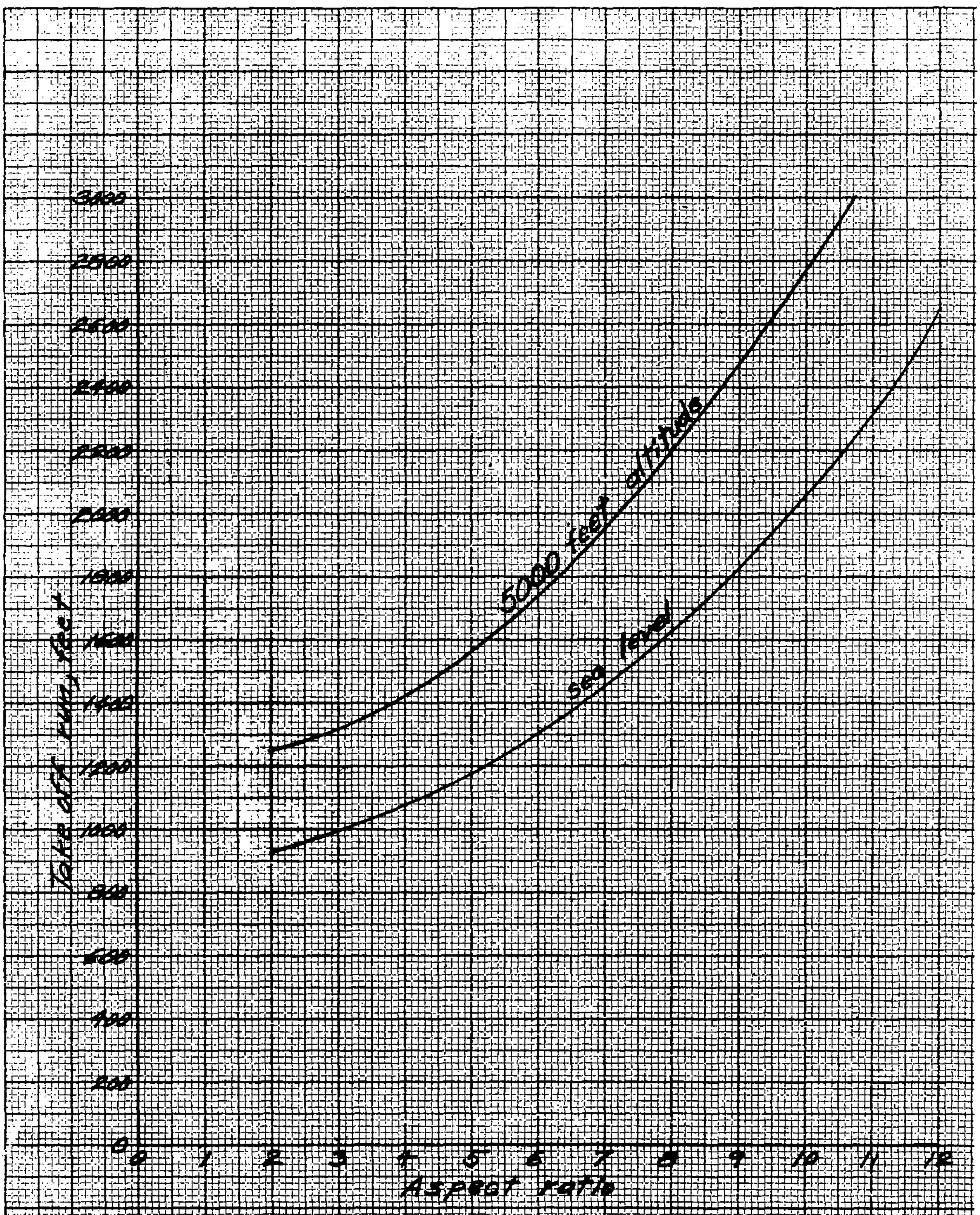

FIGURE 10(b) - EFFECT OF ASPECT RATIO ON TAKE OFF RUN

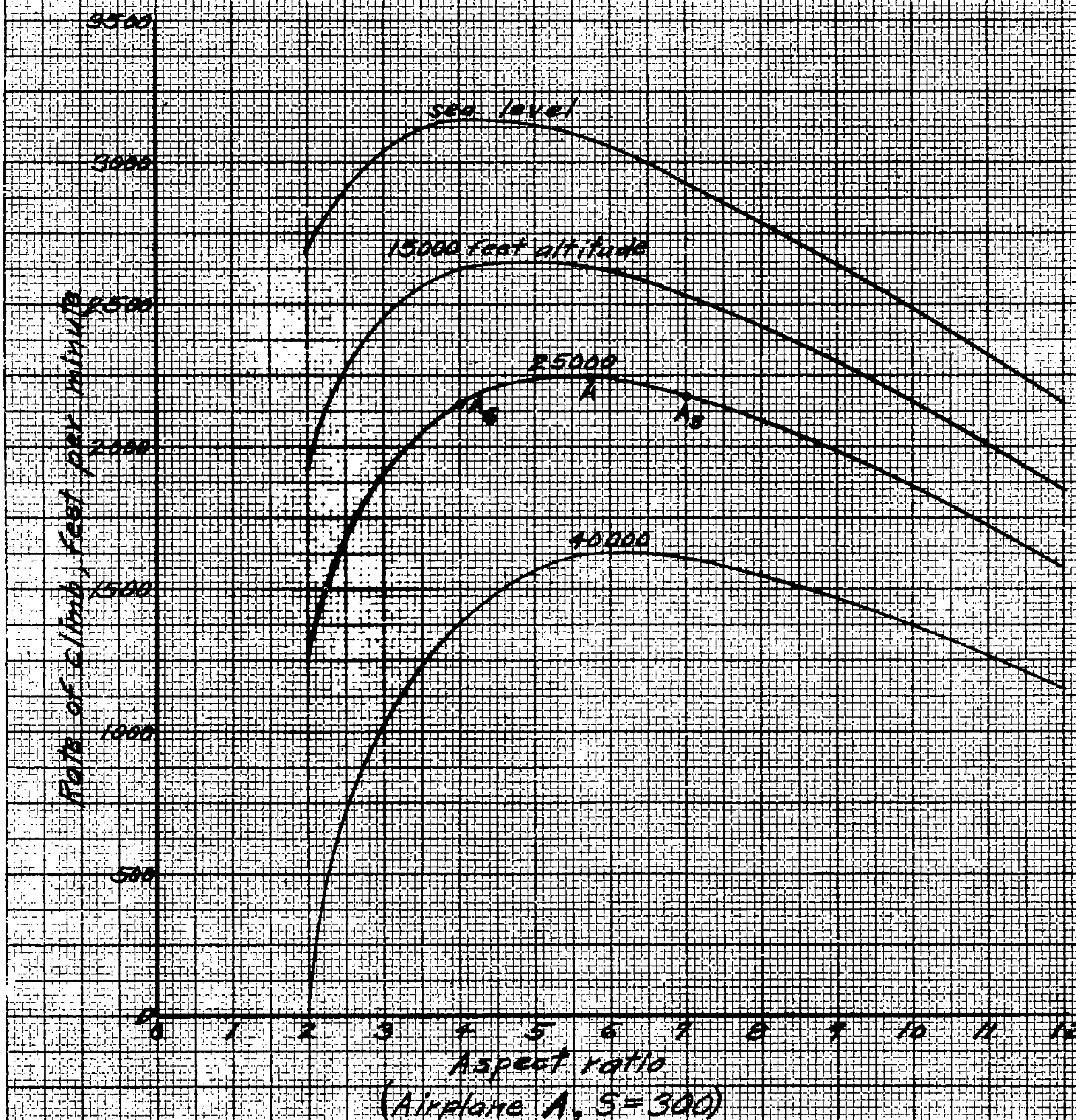

FIGURE 10(C) - EFFECT OF ASPECT RATIO ON RATE OF CLIMB

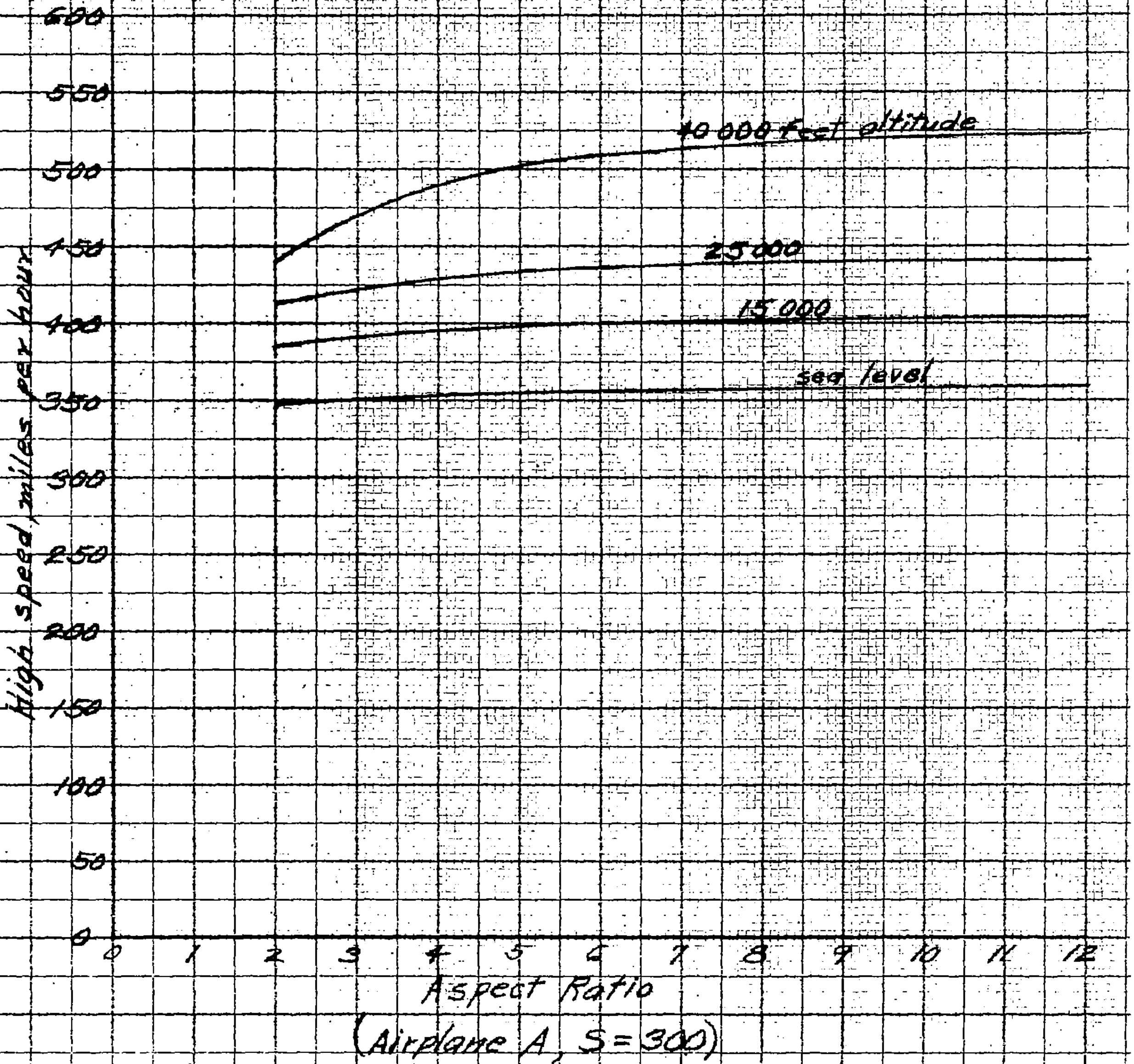

FIGURE 10(d) - EFFECT OF ASPECT RATIO ON HIGH SPEED

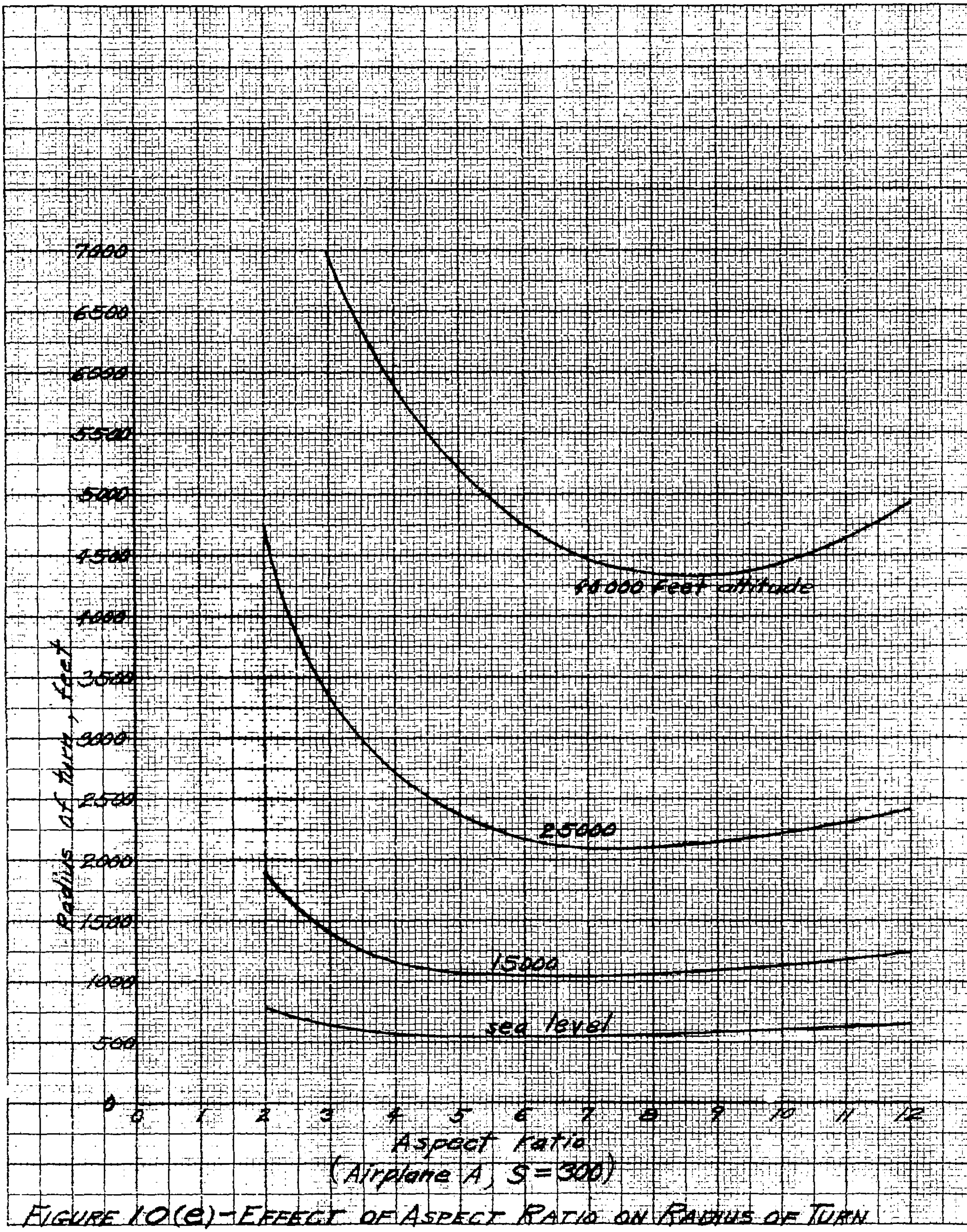

FIGURE 10(e)—EFFECT OF ASPECT RATIO ON RADIUS OF TURN

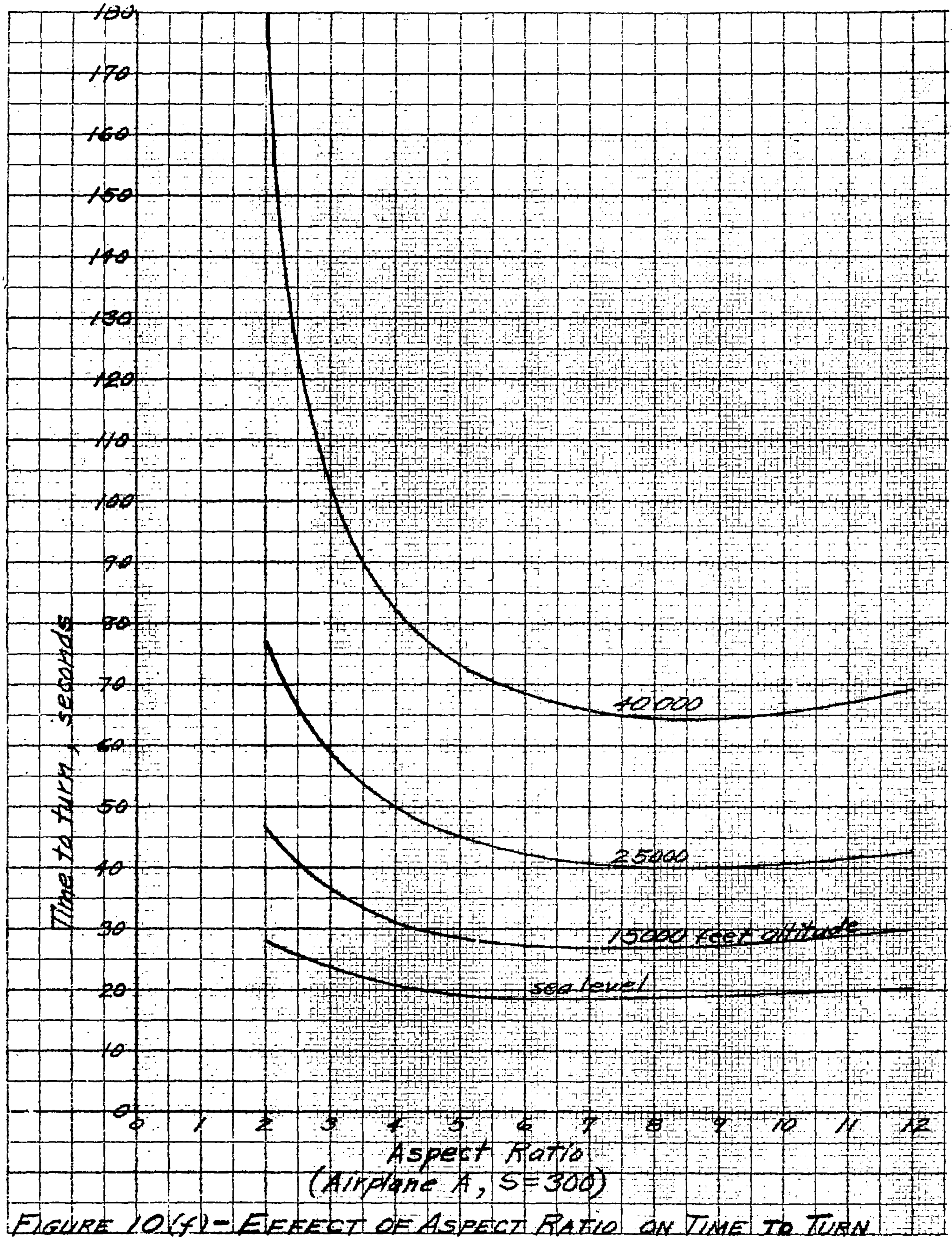

FIGURE 10(f) - EFFECT OF ASPECT RATIO ON TIME TO TURN

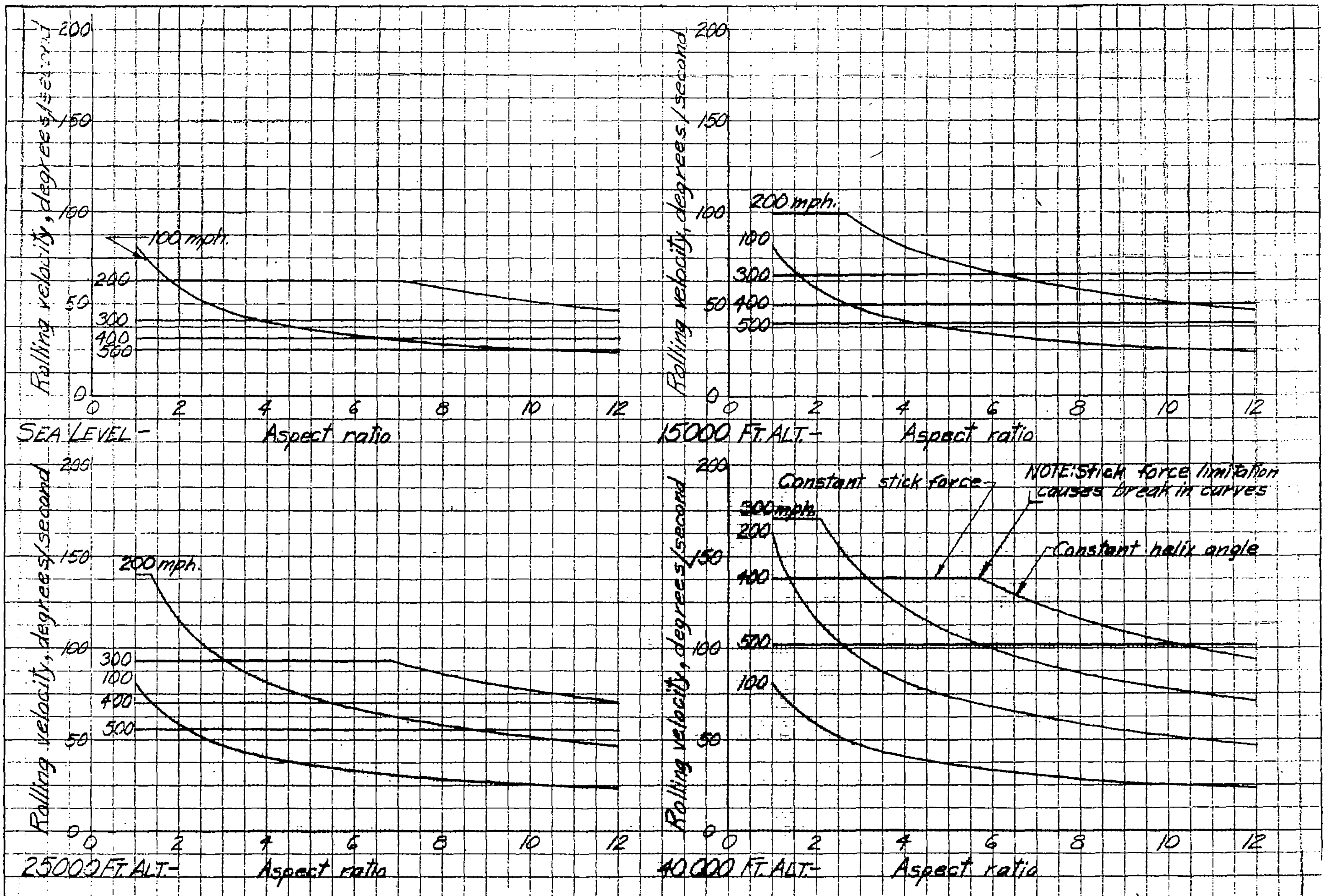

FIGURE 10(g) - EFFECT OF ASPECT RATIO ON ROLLING (AIRPLANE A, S=300, W_S/P=4.03, W & R VARY)

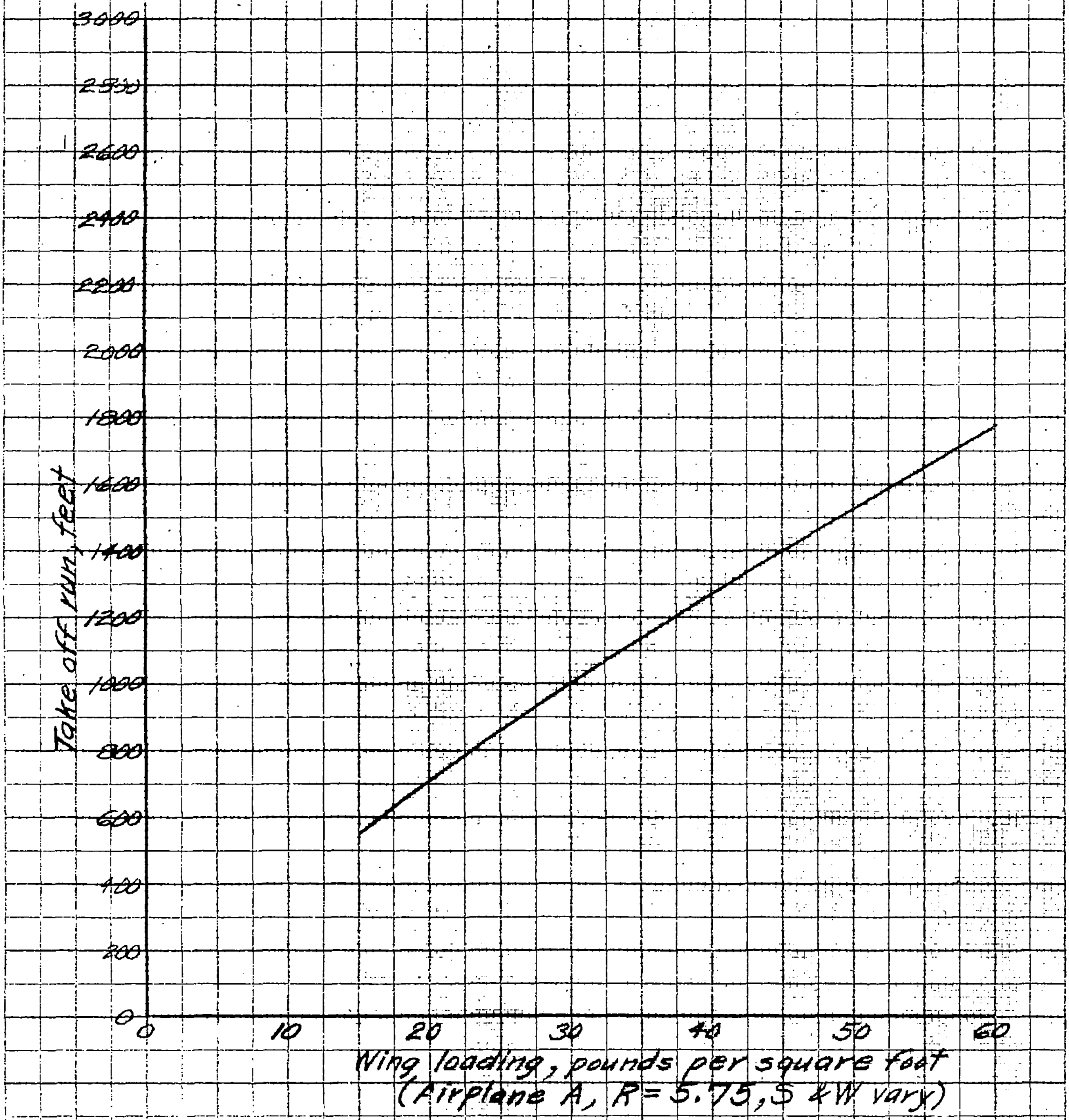

FIGURE 11 (a) - THE EFFECT OF WING LOADING ON TAKE OFF RUN

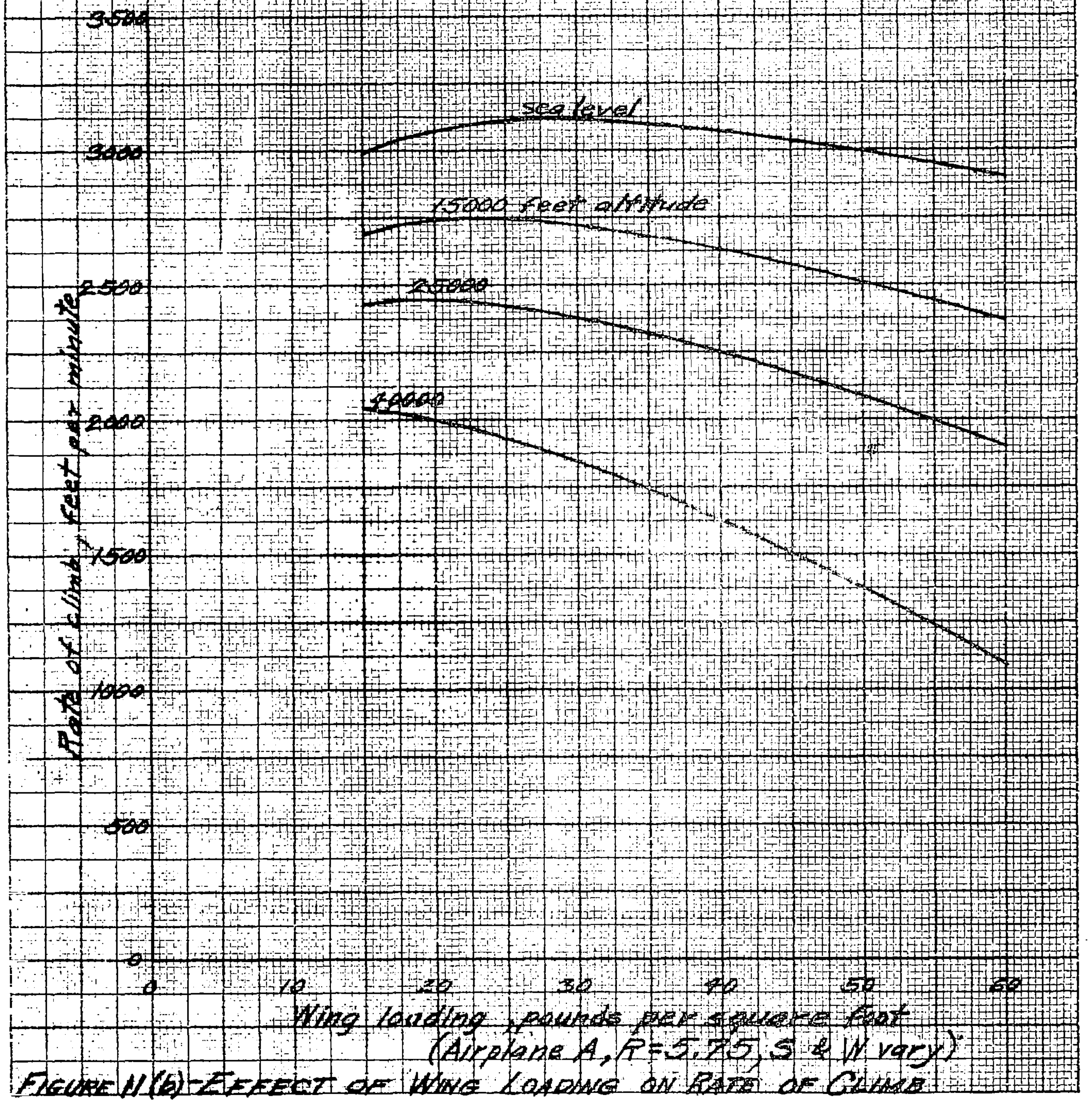

FIGURE 11(b)-EFFECT OF WING LOADING ON RATE OF CLIMB

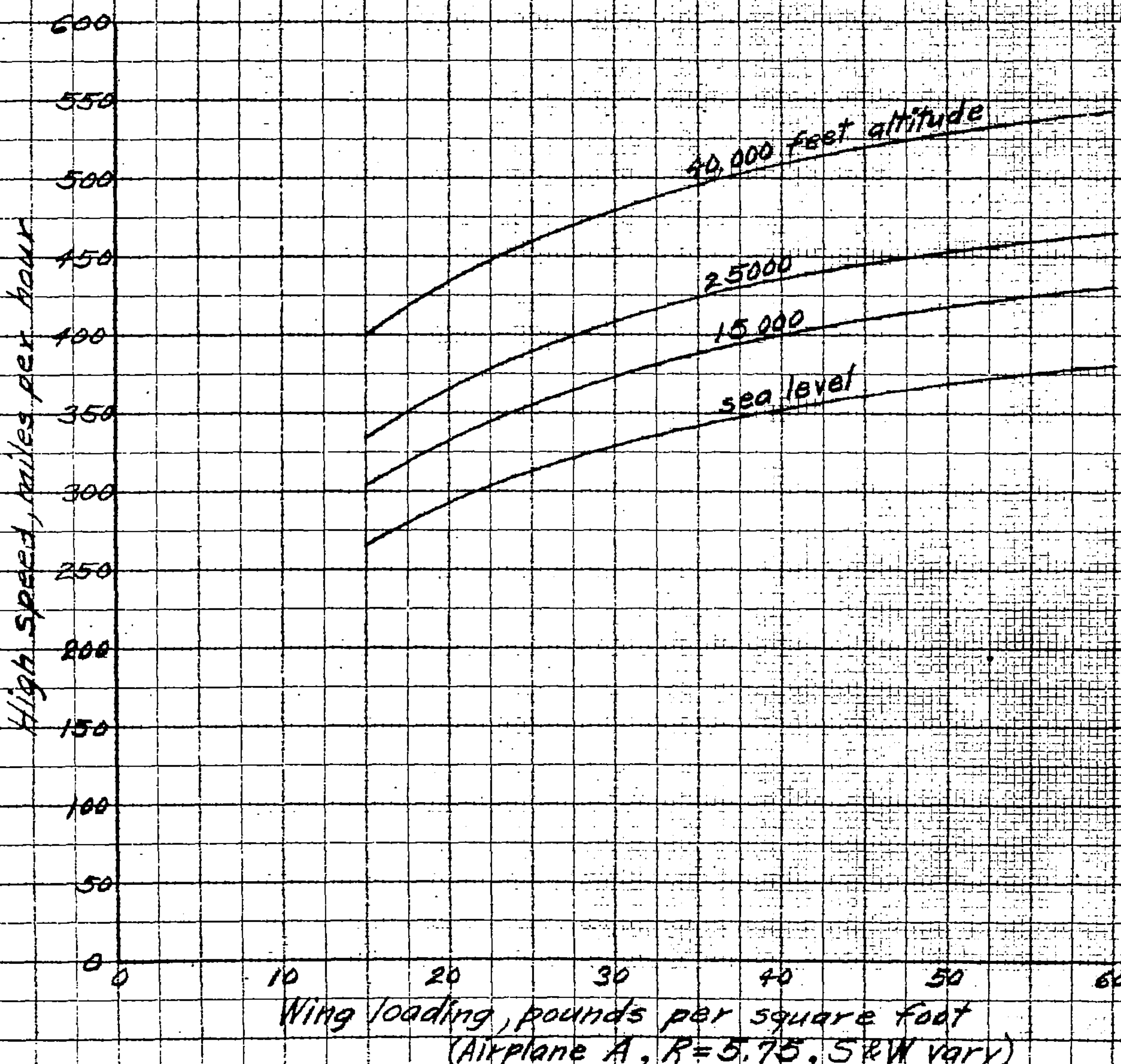

FIGURE 11(C)- EFFECT OF WING LOADING ON HIGH SPEED

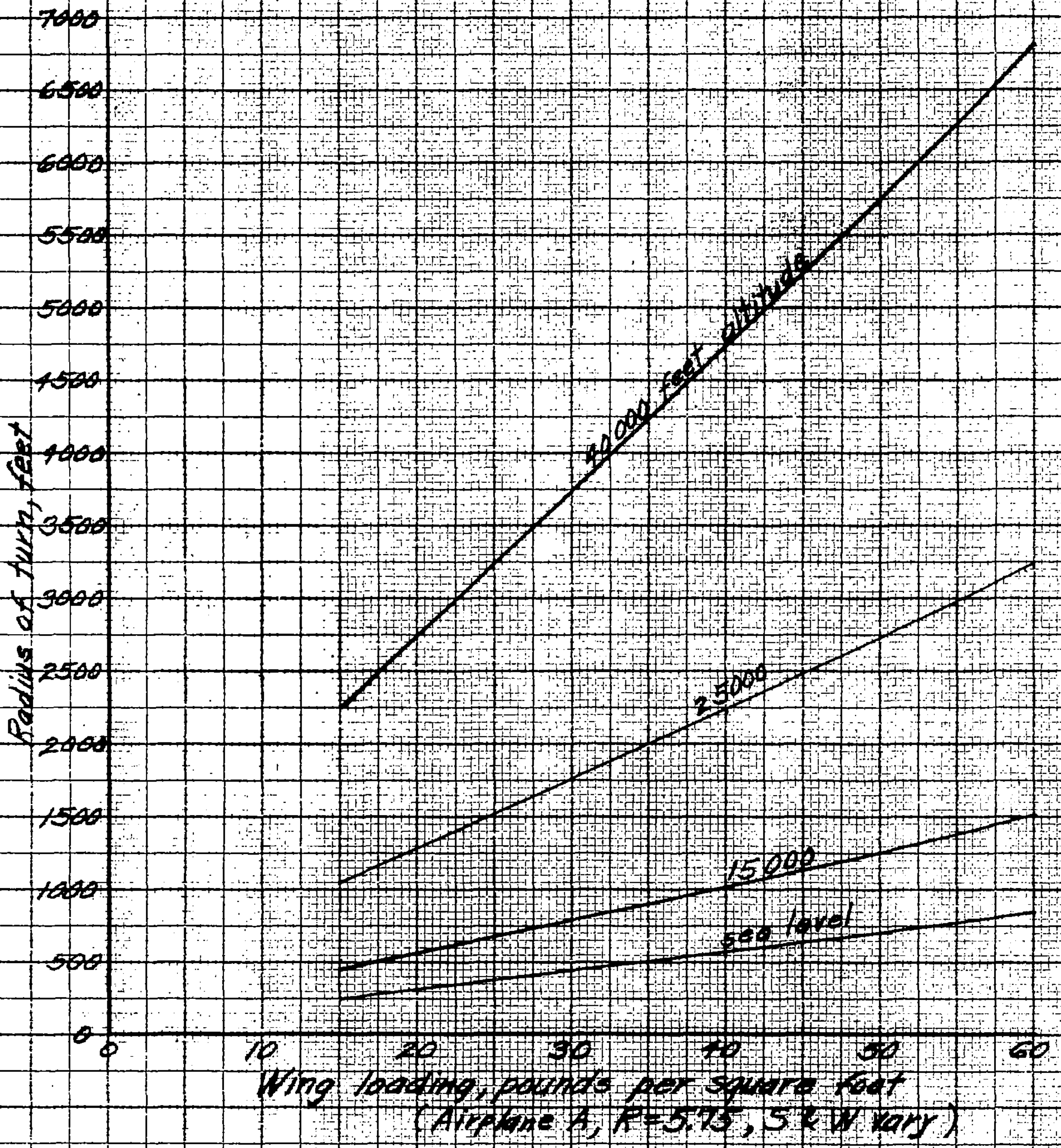

FIGURE 11 (d) EFFECT OF WING LOADING ON RADIUS OF TURN

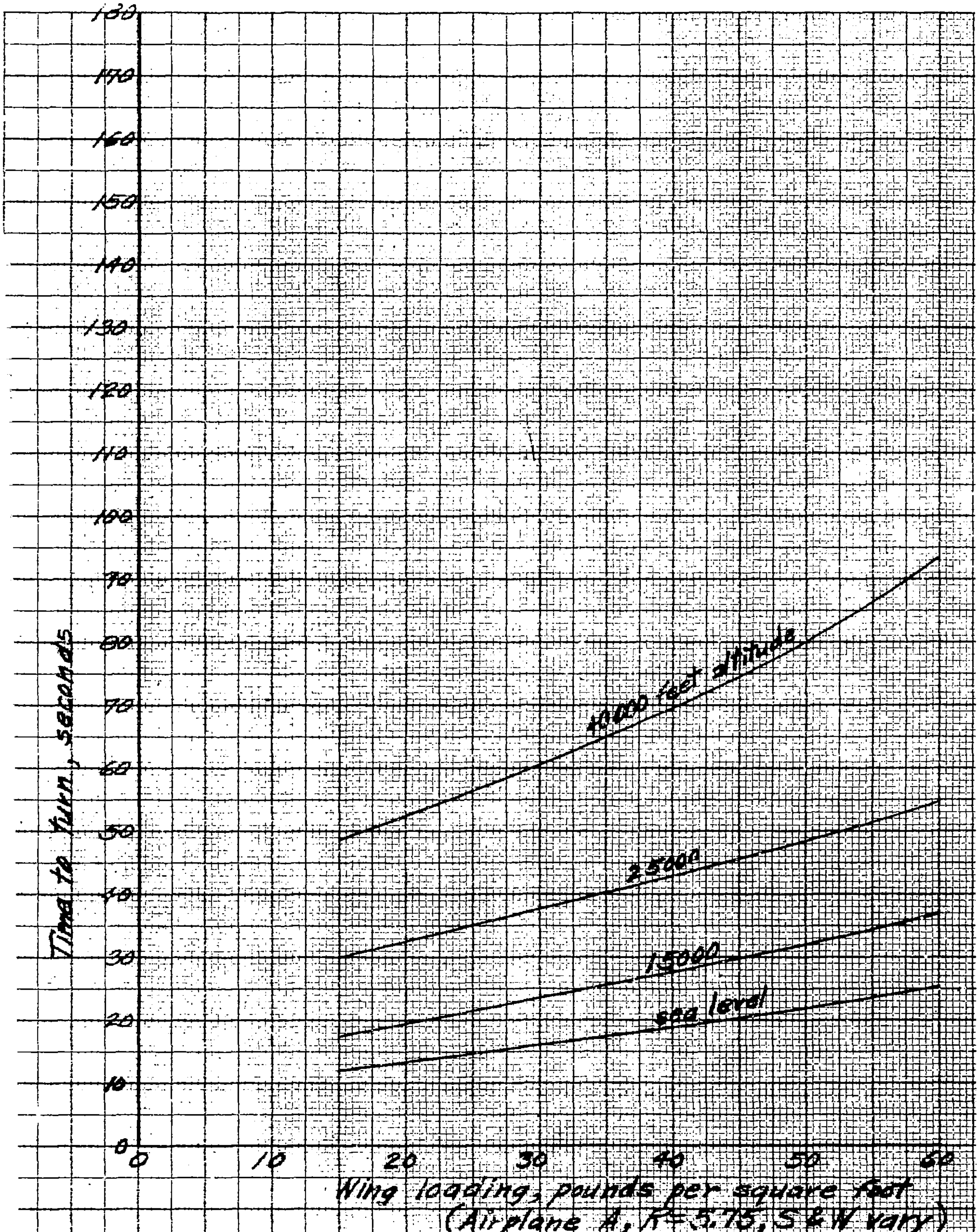

FIGURE 11 (e) - EFFECT OF WING LOADING ON TIME TO TURN

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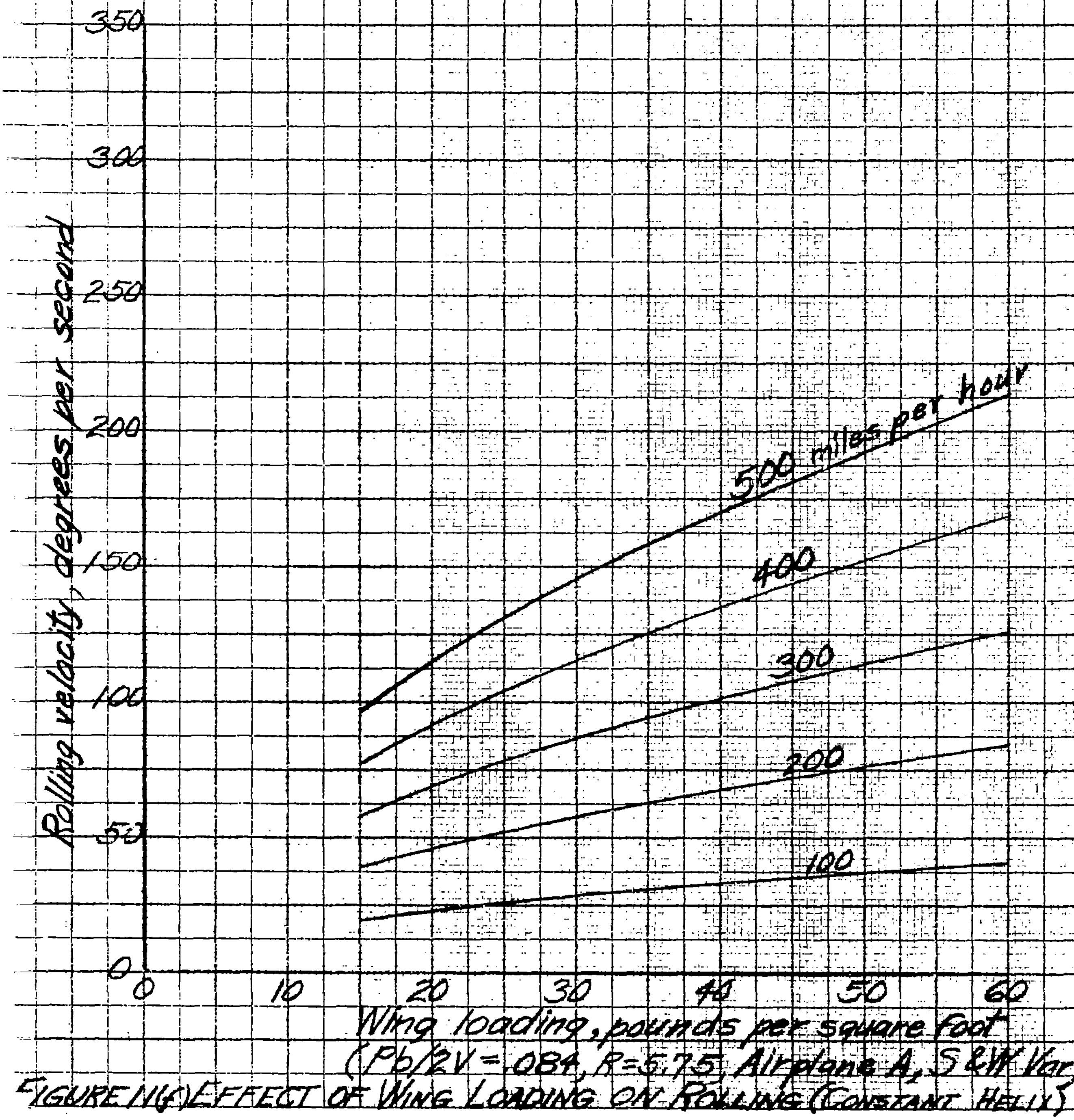

FIGURE 11(f) EFFECT OF WING LOADING ON ROLLING (CONSTANT HELIX)

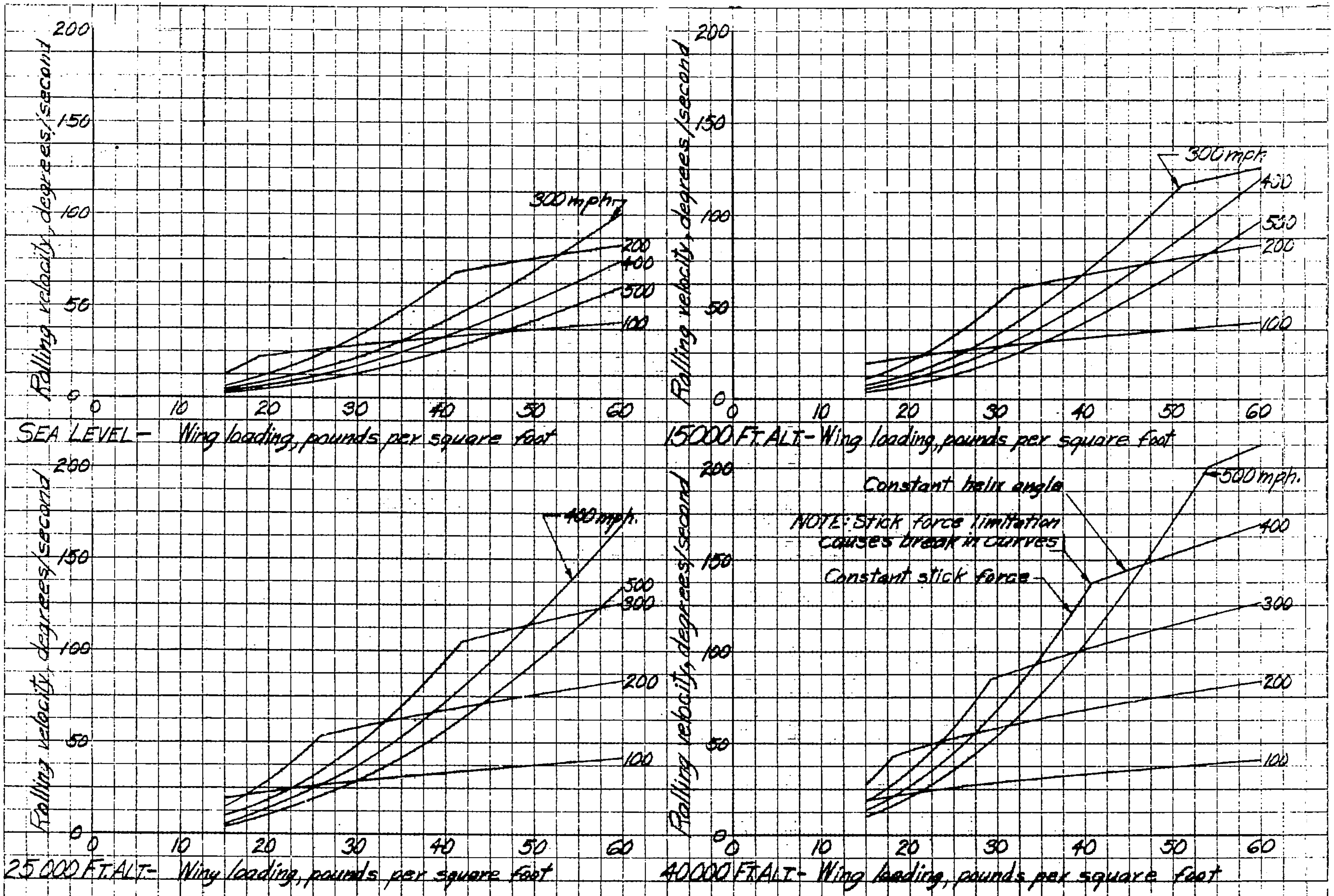

FIGURE 11(g) - EFFECT OF WING LOADING ON ROLLING (AIRPLANE A, A=5.75, W_G/P=4.05, W&S VARY, 30 LB. FORCE)